TREASURY, RISK AND FINANCE PROFESSIONALS

ACT

Examination Paper, Solutions and Examiner's Report

Paper:
Financial Mathematics and Modelling

October 2011

QUESTION 1

You are considering a potential investment of GBP 300 million in a new issue of a financial instrument, and you wish to evaluate whether the investment represents good value.

It is the end of year 2010 now and the instrument will pay out GBP 50 million per annum at the ends of Years 2013, 2014, 2015 and 2016, followed by GBP 40 million per annum at the ends of Years 2017 to 2025 inclusive.

The annual effective market yield on instruments of comparable risk is 7% for all maturities.

Required:

(a) What is the fair price of the instrument?

(4 marks)

(b) Based solely on the above, should you accept this opportunity? Explain why or why not?

(2 marks)

(Total 6 marks)

QUESTION 2

Annual effective yields on zero coupon bonds are as follows:

Maturity (years)	**1**	**2**	**3**	**4**
Yield (%)	6.0000	5.5000	5.2000	5.0000

Required:

Given the information in the table above what are the implied annual forward rates for the related periods?

(3 marks)

QUESTION 3

Your client group of companies wishes to construct a speculative bear spread option strategy using appropriate put options. The group has no existing exposures or positions in the underlying asset, nor in any related options or other derivatives.

Using the put options whose strike prices and premia are set out in the table below there are two potential means of constructing the strategy to include an option over the underlying asset with a strike price of USD 100.

Assume that the group can both buy and sell options at these prices.

Strike price USD	Put premium USD (3 month option)
90	8
100	12
110	18

Required:

(a) Explain what is meant by a "bear spread" in this context and why a trader might wish to use a bear spread strategy.

Using the graph paper supplied, illustrate your explanations with an appropriate sketch of the results of a bear spread trade, plotting gain/loss against the out-turn underlying price.

No calculations are required for this part.

(4 marks)

(b) Explain how the group could build each of the two bear spread strategies described above from the options in the table.

Calculate the net initial premium in each case.

(4 marks)

(c) What are the maximum potential profits and the maximum potential losses from each bear spread strategy?

At what out-turn underlying price or prices would the maximum profits and the maximum losses result in each case?

(4 marks)

(d) At what out-turn underlying price or prices would the profits or losses be the same for each bear spread strategy? What are the related amounts of profits or losses? Draw a sketch, using graph paper supplied, showing the two bear spread strategies on a single chart, and label the relevant breakeven points on your sketch.

(4 marks)

(e) Apart from delta, with which the group is already familiar, name four other option value sensitivity measures, i.e. 'the Greeks'. Explain each and why it is useful.

(6 marks)

(Total 22 marks)

QUESTION 4

Your Group needs to borrow GBP funds for 183 days and you are considering using either a sterling bill of exchange with a face value of GBP 5 million, or an alternative borrowing instrument that will produce the same net proceeds. Your principal relationship bank has quoted a discount rate of 4.25% for the bill, exclusive of acceptance commission of 0.125% per annum of the face value of the bill. The alternative borrowing instrument is quoted at a yield of 4.40%, inclusive of all costs.

Required:

(a) Which source of borrowing is cheaper? If you were to use the cheaper source of borrowing, what would be the saving in GBP for the 183 day period, compared with the more expensive source?

(4 marks)

A dealer in the secondary market buys a GBP 5 million face value bill at a discount rate of 4.00% with 180 days remaining maturity, and sells it 120 days later at a discount rate of 5.00%.

(b) What is the gain or loss for the dealer in GBP and what is their annual effective rate of return achieved?

Comment briefly on your result compared with the related discount rates of 4.00% and 5.00%.

(4 marks)

(Total 8 marks)

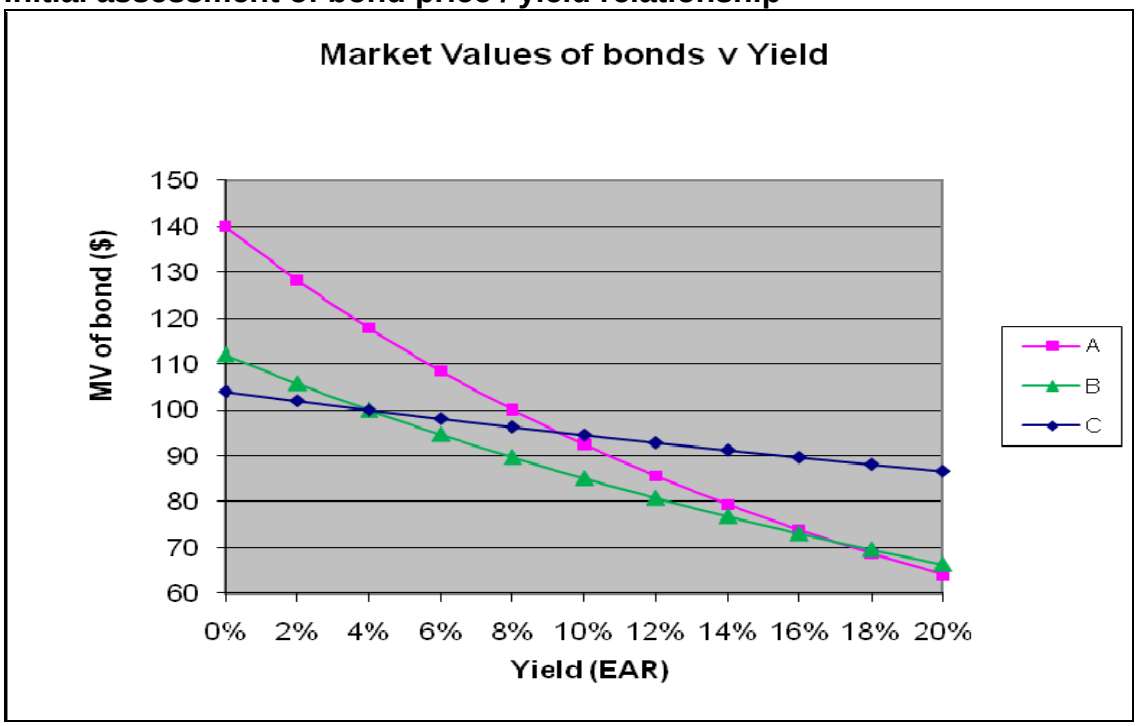
QUESTION 5

You are acting on behalf of an investment fund that is trying to assess the interest rate risk of three corporate bonds, details of which are set out below.

All three bonds pay fixed annual coupons and are redeemable at par.

	X-Ray	Yorkie	Zulu
Par value (USD)	100.00	100.00	100.00
Coupon rate	4%	4%	8%
Annual effective market yield now	5.00%	5.50%	6.00%
Remaining Maturity	1 yr	3 yrs	5 yrs

Initial assessment of bond price / yield relationship



Required:

(a) Calculate the current market price of each of the three bonds.

(4 marks)

(b) Which of the curves above labelled A, B and C represents each of the X-Ray, Yorkie and Zulu bonds respectively? Give all the factors that support your conclusion.

(6 marks)

(c) Explain the reason for:
(i) each intersection with the y-axis
(ii) each crossing point between two curves

Quantify your explanations as far as you can with examples, although no exact interpolation is required.

(6 marks)

(Total 16 marks)

QUESTION 6

You are modelling swap positions including a fixed-for-fixed EUR/FCU (Foreign Currency Unit) cross currency interest rate swap entered into two years ago at prevailing market rates when the market value of the swap was close to EUR zero.

The swap now has three years to run. Under the terms of the swap your company receives 4.3000% per annum in EUR and pays 3.2000% per annum in FCU once a year. Principal amounts of EUR 10 million and FCU 600 million will be re-exchanged at the maturity of the swap. The current market mid exchange rate is EUR/FCU 80.00.

Forward market interest rates are currently as follows:

	FCU	**EUR**
Year 0 - 1	2.0000%	3.0000%
Year 1 - 2	2.5000%	3.5000%
Year 2 - 3	3.0000%	4.0000%

Required:

(a) Calculate the current market value of the swap in EUR for your company.

(7 marks)

(b) Explain the reasons for the change in the market value of the swap between two years ago and today, quantifying your explanation with appropriate illustrative figures using your solution to part (a) above.

(3 marks)

(Total 10 marks)

QUESTION 7

You are the newly appointed Group Treasurer. Your Group has written a number of European put options over an underlying commodity in which the Group is also a market maker. You have decided to recalculate the values of the options.

The options have remaining maturities of 3 months and 6 months. Their strike price is USD 40. The underlying commodity spot price is USD 35. The risk free rate of return is 5.00% continuously compounded per year. You decide to value the options using appropriate risk-neutral binomial trees, assuming the underlying commodity price jumps exactly once by 20% either up or down per 3 month period.

Required:

(a) Use a one period risk neutral binomial tree to value the 3 month put options.

(5 marks)

(b) Use a two period risk neutral binomial tree to value the 6 month put options.

(5 marks)

(c) (i) Use the put-call parity relationship to calculate the value of a comparable 3 month call option on the commodity, based on the put value you have calculated in part (a).

(ii) Explain why the value you have calculated appears reasonable.

(5 marks)

(Total 15 marks)

QUESTION 8

You are considering constructing an investment portfolio by combining different proportions of two assets: Asset A and Asset B.

Asset A has an expected return of 5% per annum and a standard deviation of expected returns of 7%. Asset B has an expected return of 12% and a standard deviation of expected returns of 16%.

Your internal research has shown, reliably, that there is no connection at all between the historical returns of the two assets.

You want to make sure that your portfolio has as good a risk / return performance as is possible given the choice of assets in which to invest.

Required:

(a) Explain how and why portfolio risk varies as the composition of the portfolio varies from 0% Asset A (i.e. 100% Asset B) to 100% Asset A (i.e. 0% Asset B). Use calculations and sketch a diagram in your answer book to illustrate your explanation.

(3 marks)

(b) Using the information above and drawing appropriate deductions, draw a diagram in your answer book charting the relationship for the same range of portfolios. Use portfolio risk as the horizontal axis and portfolio expected return as the vertical axis.

Indicate and briefly explain any features of the chart that you think relevant.

(3 marks)

(Total 6 marks)

QUESTION 9

You are a Treasury Analyst with Overseas Investors PLC, an investment fund. The fund has foreign currency assets of CHF 150 million and USD 300 million.

Spot exchange rates are:

GBP/CHF	2.2000
GBP/USD	1.5500

You have obtained the following information about the daily standard deviations and correlation coefficient of exchange rate movements:

Daily Standard Deviations	GBP/CHF	0.70%
	GBP/USD	0.40%

Correlation coefficient GBP/CHF vs GBP/USD + 0.45

Required:

(a) Use the information above to calculate the daily Value at Risk (VaR) in GBP at the 95% confidence level of the fund's exposure to each of the currencies on an individual basis.

(4 marks)

(b) Calculate the 10 day correlated VaR in GBP at the 99% confidence level of the combined exposure to the portfolio containing both currencies.

(4 marks)

(c) Explain the key assumptions underlying your VaR calculations in parts (a) and (b), describing how VaR is sensitive to them.

(6 marks)

(Total 14 marks)

FORMULAE

Continuous compounding

$$FV = PV \times e^{rT}$$

$$PV = FV \times e^{-rT}$$

Present value of an annuity; Annuity Factor

$$PV = A_1 \text{ x } AF_{(r,n)}$$

$$AF_{(r,n)} = \frac{1}{r} \text{x } [1-(1+r)^{-n}]$$

Sample variance

$$Var[X] = \frac{1}{n}\sum_{i=1}^{n}(x_i - E[X])^2$$

Estimated population variance

$$Var[X] = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - E[X])^2$$

Covariance

$$\sigma_{xy} = E[(x-\mu_x)(y-\mu_y)]$$

Coefficient of correlation

$$\rho_{xy} = \frac{\sigma_{xy}}{\sigma_x \sigma_y}$$

Macaulay Duration

$$\text{Duration (D)} = \frac{\text{Sum (PV} \times \text{t)}}{\text{Sum (PV)}}$$

Modified duration

$$D_{MOD} = \frac{\text{Macaulay Duration(D)}}{[1+r]}$$

Convexity

$$\frac{\text{Sum}(PV \times t \times (t+1))}{\text{Sum}(PV)}$$

Modified convexity

$$C_{MOD} = \frac{\text{Macaulay Convexity(C)}}{[1+r]^2}$$

Option pricing: single period binomial model (probability of an uptick)

$$p = \frac{e^{rT} - d}{u - d}$$

Option pricing: Black Scholes model

$$C = S_0 \times N(d_1) - X \times e^{-rT} \times N(d_2)$$

$$P = X \times e^{-rT} \times N(-d_2) - S_0 \times N(-d_1)$$

$$d_1 = \frac{\ln\left(S_0 / X\right) + \left(r + \sigma^2 / 2\right) \times T}{\sigma \times \sqrt{T}}$$

$$d_2 = \frac{\ln\left(S_0 / X\right) + \left(r - \sigma^2 / 2\right) \times T}{\sigma \times \sqrt{T}} = d_1 - \sigma \times \sqrt{T}$$

Option pricing: put-call parity relationship

$$S_0 + P - C = Xe^{-rT}$$

VaR

$$\text{VaR} = Z \times \sigma \times \text{Exposure}$$

VaR holding period adjustment

$$\sigma_{t2} = \sigma_{t1} \times \sqrt{\frac{t_2}{t_1}}$$

Correlated VaR

$$\text{VaR}_{AB} = \sqrt{\text{VaR}_A^{\,2} + \text{VaR}_B^{\,2} + \left(2 \times \rho_{AB} \times \text{VaR}_A \times \text{VaR}_B\right)}$$

STANDARDISED NORMAL DISTRIBUTION TABLE

Cumulative Distribution Function for the Standard Normal Random Variable [N(x)] where $x \geq 0$.

The table shows values of N(x) for $x \geq 0$. The table can be used with interpolation.

For example:

$$N(0.4245) = N(0.42) + 0.45 \times [N(0.43) - N(0.42)] = 0.663 + 0.45 \times (0.666 - 0.663) = 0.664$$

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	0.500	0.504	0.508	0.512	0.516	0.520	0.524	0.528	0.532	0.536
0.1	0.540	0.544	0.548	0.552	0.556	0.560	0.564	0.567	0.571	0.575
0.2	0.579	0.583	0.587	0.591	0.595	0.599	0.603	0.606	0.610	0.614
0.3	0.618	0.622	0.626	0.629	0.633	0.637	0.641	0.644	0.648	0.652
0.4	0.655	0.659	0.663	0.666	0.670	0.674	0.677	0.681	0.684	0.688
0.5	0.691	0.695	0.698	0.702	0.705	0.709	0.712	0.716	0.719	0.722
0.6	0.726	0.729	0.732	0.736	0.739	0.742	0.745	0.749	0.752	0.755
0.7	0.758	0.761	0.764	0.767	0.770	0.773	0.776	0.779	0.782	0.785
0.8	0.788	0.791	0.794	0.797	0.800	0.802	0.805	0.808	0.811	0.813
0.9	0.816	0.819	0.821	0.824	0.826	0.829	0.831	0.834	0.836	0.839
1.0	0.841	0.844	0.846	0.848	0.851	0.853	0.855	0.858	0.860	0.862
1.1	0.864	0.867	0.869	0.871	0.873	0.875	0.877	0.879	0.881	0.883
1.2	0.885	0.887	0.889	0.891	0.893	0.894	0.896	0.898	0.900	0.901
1.3	0.903	0.905	0.907	0.908	0.910	0.911	0.913	0.915	0.916	0.918
1.4	0.919	0.921	0.922	0.924	0.925	0.926	0.928	0.929	0.931	0.932
1.5	0.933	0.934	0.936	0.937	0.938	0.939	0.941	0.942	0.943	0.944
1.6	0.945	0.946	0.947	0.948	0.949	0.951	0.952	0.953	0.954	0.954
1.7	0.955	0.956	0.957	0.958	0.959	0.960	0.961	0.962	0.962	0.963
1.8	0.964	0.965	0.966	0.966	0.967	0.968	0.969	0.969	0.970	0.971
1.9	0.971	0.972	0.973	0.973	0.974	0.974	0.975	0.976	0.976	0.977
2.0	0.977	0.978	0.978	0.979	0.979	0.980	0.980	0.981	0.981	0.982
2.1	0.982	0.983	0.983	0.983	0.984	0.984	0.985	0.985	0.985	0.986
2.2	0.986	0.986	0.987	0.987	0.987	0.988	0.988	0.988	0.989	0.989
2.3	0.989	0.990	0.990	0.990	0.990	0.991	0.991	0.991	0.991	0.992
2.4	0.992	0.992	0.992	0.992	0.993	0.993	0.993	0.993	0.993	0.994
2.5	0.994	0.994	0.994	0.994	0.994	0.995	0.995	0.995	0.995	0.995
2.6	0.995	0.995	0.996	0.996	0.996	0.996	0.996	0.996	0.996	0.996
2.7	0.997	0.997	0.997	0.997	0.997	0.997	0.997	0.997	0.997	0.997
2.8	0.997	0.998	0.998	0.998	0.998	0.998	0.998	0.998	0.998	0.998
2.9	0.998	0.998	0.998	0.998	0.998	0.998	0.998	0.999	0.999	0.999
3.0	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999
3.1	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999
3.2	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999
3.3	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
3.4	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

Suggested Solutions for October 2011

General

HEALTH WARNINGS

Level of detail required

In order to serve as a resource for future students, many of these solutions are set out in more detail than would be expected from candidates under exam conditions.

The best guide to the expected level of detail in the exam is what a well-prepared candidate could reasonably be expected to produce in the time implied by the marks in the question. We recommend allocating 1.5 minutes per mark x 100 marks = 150 minutes, to allow yourself 30 minutes total reading, planning and contingency time.

Solutions are not totally comprehensive

Solutions set out here are not totally comprehensive of all of the relevant points which could be made in a fuller discussion of the selected topics.
Additional valid and relevant points - if clearly set out - will always be credited in your exam, even if they are not incorporated into the published solutions.

Alternative solutions

Any clear route to a valid solution will be awarded full marks. You do not need to use the same calculation methods as those illustrated.

The answers illustrated are not necessarily the only ones possible. For reasons of space, alternative valid assumptions, methods and answers are not normally set out in the published solution, but they are fully credited in the exam.

Slightly different numerical solutions may also be calculated, depending on the rounding of intermediate figures.

Explanatory & illustrative diagrams

You are strongly encouraged to incorporate diagrams within your solutions to illustrate and clarify an explanation where relevant. For reasons of space, these solutions are not comprehensive of all of the possible relevant diagrams. Candidates who produced relevant diagrams were awarded marks for doing so.

QUESTION 1

A future-starting annuity is the difference between two simple annuities.

PV of cash flows at years 3, 4, 5 & 6 = PV(flows 1, 2 ,3, 4, 5 & 6) less PV(flows 1 & 2)

(a)

1. Value of £50m payments from T3 to T6:
This is the value of a 6 period annuity less a 2 period annuity

PV(flows 3-6) = PV(flows 1-6) *less* PV(flows 1-2)
= 50,000 x [AF(0.07, 6yrs) - AF(0.07, 2yrs)]
= 50,000 x [1/0.07 x (1 - 1.07^{-6}) - 1/0.07 x (1 - 1.07^{-2})]
= 50,000 x [4.76654 – 1.80801]
= 50,000 x [2.95852]
= GBP 147.926 m

2. Value £40 payments from T7 to T15:
PV(flows 7-15) = PV(flows 1-15) *less* PV(flows 1-6)
= 40,000 x [AF(0.07, 15yrs) - AF(0.07,6yrs)]
= 40,000 x [1/0.07 x (1 - 1.07^{-15}) - 1/0.07 x (1 - 1.07^{-6})]
= 40,000 x [9.10791-4.76654]
= 40,000 x [4.34137]
= GBP 173.665 m

3. Total present value of instrument:
= 147.923 + 173.665
= GBP 321.581 m

(b)

The opportunity should be accepted because the PV of future inflows of GBP 321.581 million exceeds the initial outflow of £300 million.

The NPV of the proposal is greater than zero (321.581 – 300 = 21.581).
Therefore accepting the opportunity to purchase the instrument would result in increased value of £21.581 million.

QUESTION 2

The yield curve is falling for longer maturities, so the forward yield curve will be lower than the zero coupon curve for the related maturities.

Using the no-arbitrage relationship between the forward interest rate and the zero coupon rates for the start and end of the forward periods:

0-1 year forward rate:
= rz1 = 0.06/1
= **6.0000% EAR**

1-2 years forward rate:
= $(1+rz2)^2 /(1+rz1) – 1$
= $(1.055)^2 /(1.060) – 1$
= 0.050024 = **5.0024% EAR**

Similarly:
2-3 years forward rate:
= $(1.052)^3 /(1.055)^2 - 1$
= 0.046026 = **4.6026% EAR**

3-4 years forward rate:
= $(1.050)^4 /(1.052)^3 - 1$
= 0.044023 = **4.4023% EAR**

[The forward rates are lower than the zero coupon rates when the yield curve is falling, as expected.]

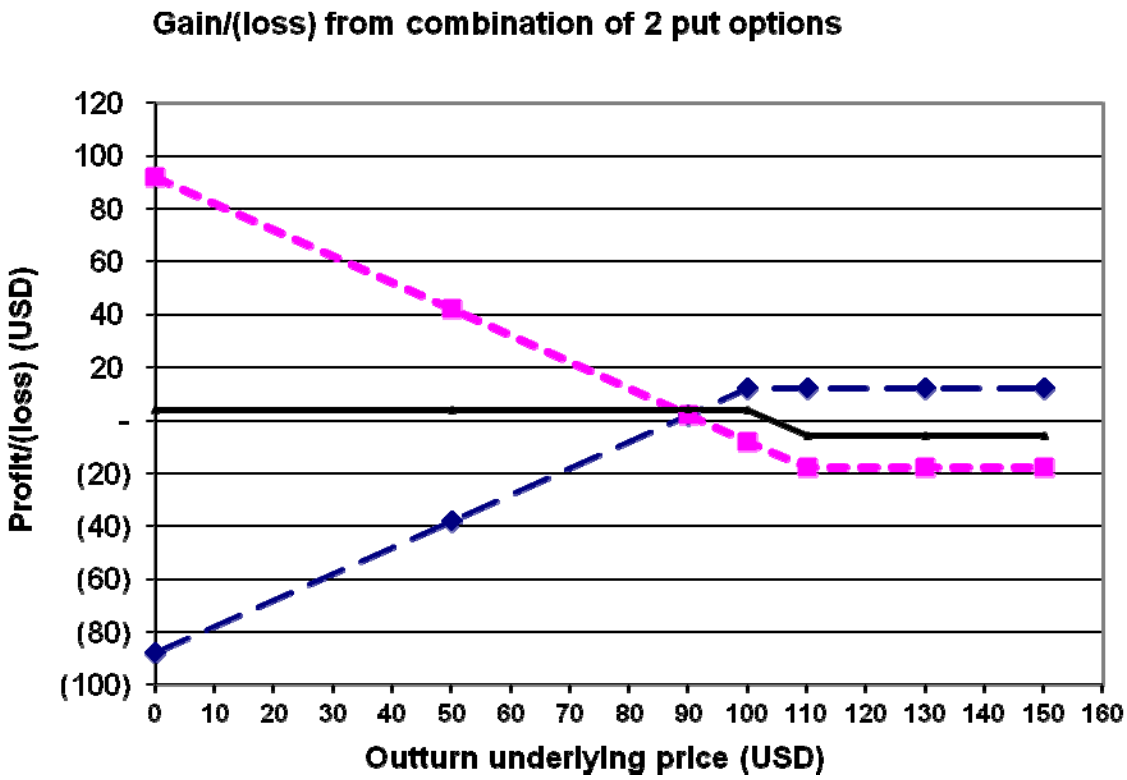
QUESTION 3

(a)

A bear spread is a speculation strategy on the price of the underlying falling. If the speculator knew for certain that the underlying price would fall, then a put option would be a better deal – giving a greater gain as prices fall – but the premium would be high. Thus the bear spread is a put option where the dealer has sold some of the potential gain in order to reduce the cost of the premium.

The bear spread could be constructed using either call options or put options.

The diagram is shown below.



Key points:

- Flat payoff well above and below the expected changes in the underlying.
- Showing a gain at low underlying prices and a loss at higher underlying prices.
- The slope between the two flat payoffs is a negative slope showing a gain accumulating as underlying price falls.

(b)

The strategy can be constructed by buying a put with a higher strike and selling a put with a lower strike – as indicated on the chart above. In this case, with the requirement to include an option with strike price of USD 100, that could be achieved by either buying the put with strike 110 and selling the put with strike100, or buying the put with strike 100 and selling the put with 90.

Combination 1:

buy strike 110, sell strike 100

net premium = 18-12 = 6 paid out, having bought the more expensive option.

Combination 2:

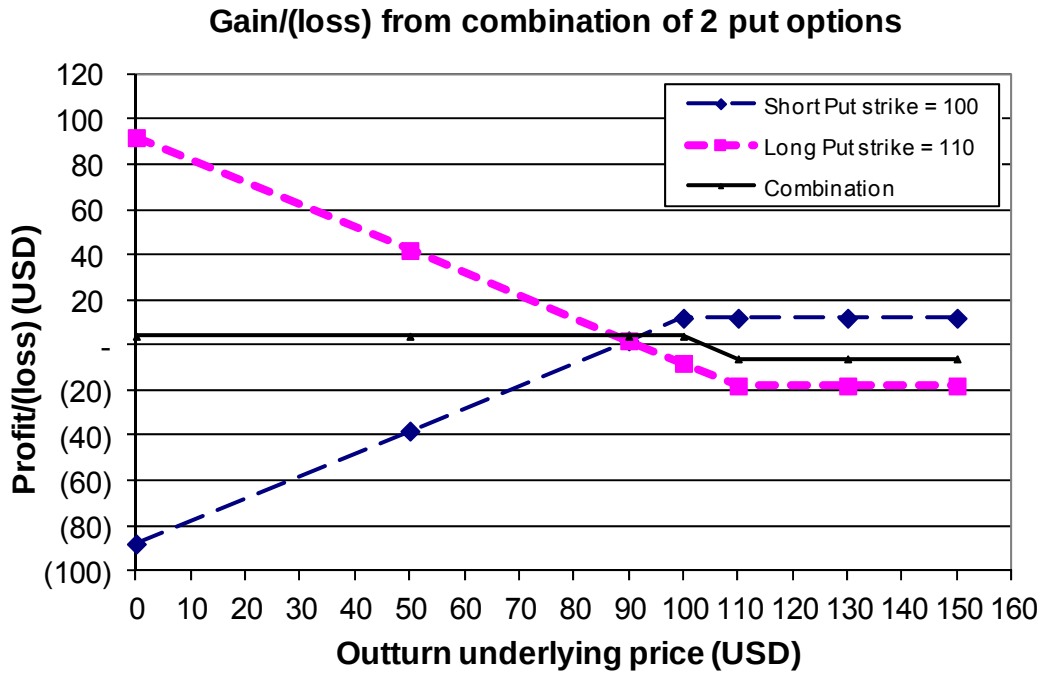
buy strike 100, sell strike 90

net premium = 12-8 = 4 paid out, having bought the more expensive option

(c)
It is easiest to draw the diagram and then either calculate or read off what is required.

[It is expected that most candidates would calculate while using the diagram as a background.]

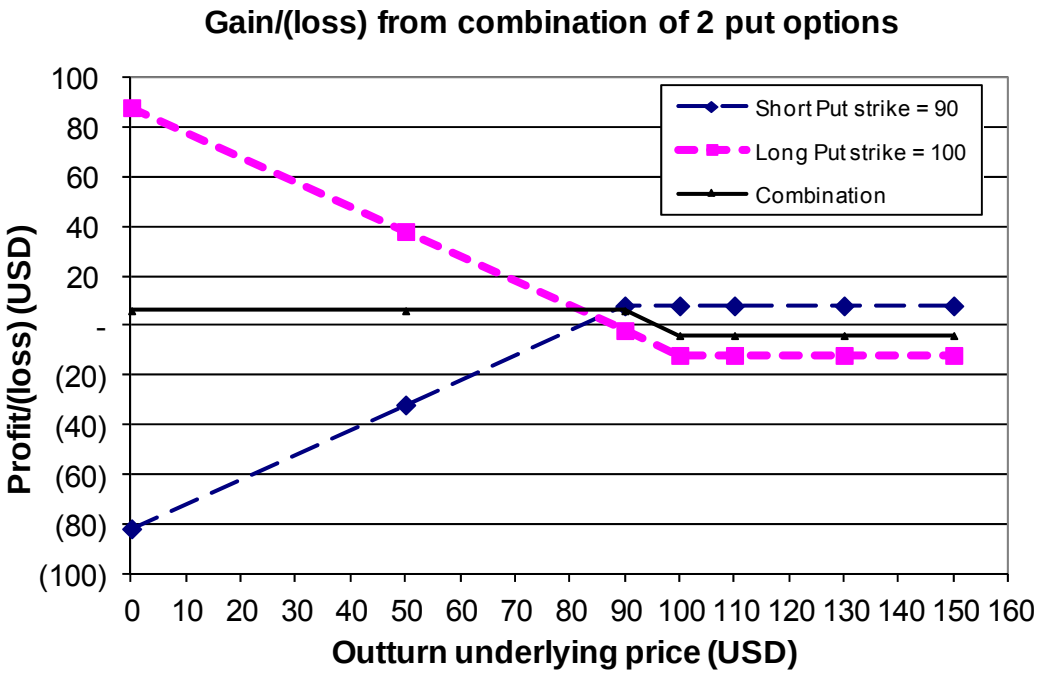
Combination 1:



When underlying =100, the bought option (strike 110) will show a gain of 10 less initial premium of 18 = -8. The sold option (strike 100) is at its strike price, so will show just a gain of the premium received, 12. The net position = -8 +12 = +4.

Max gain (+4) at outturn underlying less than or equal to 100, max loss (initial premium -6) when outturn underlying greater than or equal to 110.

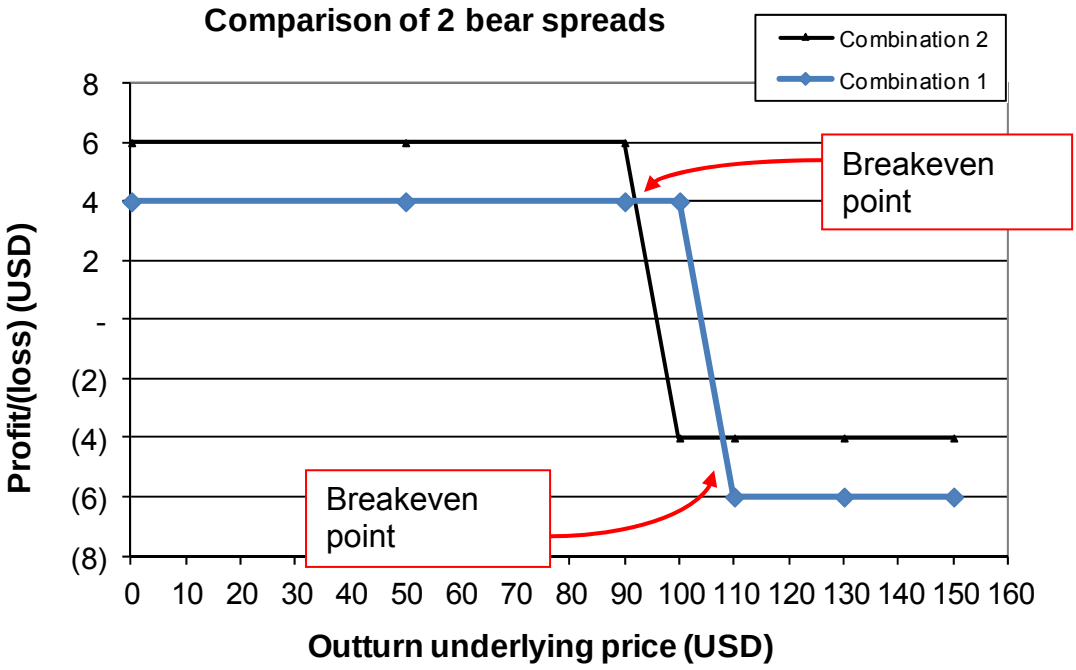
Combination 2:



When underlying =90, the bought option (strike 100) will show a gain of 10 less initial premium of 12 = -2. The sold option (strike 90) is at its strike price, so will show just a gain of the premium received, 8. The net position = -2 +8 = +6.

Max gain (+6) at outturn underlying less than or equal to 90, max loss (initial premium -4) when outturn underlying greater than or equal to 100.

(d)



The points where the results are the same can be seen on the diagram above – where both have a result of either +4 or -4. This occurs when:

- +4 outcome when underlying is 90+2 =92. This is because outcome is 2 less than the max outcome for combination 2, the underlying must therefore be 2 greater than the lower strike – i.e. 92

- -4 outcome when underlying is 110-2=108. This is because outcome is 2 better than the max loss for combination 1, -6. The underlying must therefore be 2 lower than the higher strike – i.e. 108.

(e)

The Greeks:
Theta – the rate of change of the option value as the time to maturity changes. It is useful because it indicates how rapidly the option value will change over the time that it is held. For example, if the option is in the money and there is a short time to go before expiry, it will help to decide whether to sell the option back or to wait until closer to expiry before deciding. For call options, value always increases with more time. For put options the relationship is more complex.

Rho – the change in the option value as interest rates change. Strictly, it is the change in the continuously compounded rate of interest – but the overall yield environment is the real driver. It is useful because it helps to make the decision on whether to buy an option today, wait till an anticipated rate change or deal before an anticipated rate change. Increasing rates increase the value of call options but decrease the value of put options. The main cause is the impact of the risk free rate on the expected future impact on underlying prices, drifting upwards at the risk free rate.

Vega – the rate of change of the option value as volatility changes. Again, an unstable environment will contribute to higher option values – making the choice of an option as a hedging instrument harder to justify. Higher volatility might mean a 'better value' option – but that relates to potential gains rather than hedging outcomes. An extra value point would be the importance of expected volatility rather than historic volatility – the measure of volatility used in option pricing is normally the expected level of volatility and it is this that influences prices more than the historic level which can be quantified more precisely.

Gamma – this is a second derivative whereas all of the others are first derivatives. It measures the rate of change of delta as underlying prices change. Delta measures the rate at which option values change as underlying prices change – gamma measures the rate at which delta changes as underlying process change. Probably used more by traders than corporates, this measure is still useful in that it tells us whether we are in an area where option prices will change rapidly or slowly as the price of the underlying changes.

QUESTION 4

(a)
Bill of exchange:
Face value repayable £5,000,000
Less: discount = 0.0425 x 183/365 x 5,000,000 = 106,541.10
Less: commission = 0.00125 x 183/365 x 5,000,000 = 3,133.56
So net proceeds received = £4,890,325.34

Periodic Cost = (5,000,000 – 4,890,325.34) / 4,890,325.34 = 0.022427 (2.243%)
EAR = $(1+.022427)^{(365/183)}$ -1 = 4.5230%

Yield instrument:
Same net receipt =£ 4,890,325.34
Interest payable = 0.0440 x 183/365 x £4,890,325.34 = 107,881.92
So amount repayable = £ 4,890,325.34 + 107,881.92 = 4,998,207.26
It is clear that this is cheaper already.
Periodic cost = 107,881.92 / 4,890,325.34 = 0.02206
EAR = $(1+0.02206)^{(365/183)}$ -1 = 4.4483%

Saving with cheaper source
The **yield instrument is cheaper**.
For the same net receipt: difference = 5,000,000.00 – 4,998,207.26
= £1,792.74

(b)
Purchase cost
Face value = £5,000,000
Less discount = 0.0400 x (180/365) x 5,000,000 = 98,630.14
Purchase cost = £5m less 98,630.14 = 4,901,369.86
Sale proceeds
Remaining maturity = 180 - 120 = 60 days
Face value = £5,000,000
Less discount = 0.0500 x (60/365) x 5,000,000 = 41,095.89
Sale proceeds = £5m less 41,095.89 = 4,958,904.11
Periodic Return = (Sale proceeds / Purchase cost) – 1
= (4,958,904.11/4,901,369.86) – 1
= 0.011738

The gain for the dealer is £4,958,904.11 - £4,901,369.86 = **£57,534.25**

EAR = $(1+ 0.011738)^{(365/120)}$ – 1
= 3.6134%

Comments
The overall return earned by the dealer has been reduced by the rise in the discount rate between the purchase date and the date of sale. If the dealer's cost of funds exceeds the amount earned of 3.6134% EAR, then the net returns after funding costs would be negative.

The dealer was effectively taking a risk on the interest rate: if rates had fallen he would have earned a good return – but rates rose instead and he made a poor return. This is one aspect of interest rate risk.

If this risk was to be avoided, a corporate would normally invest in an instrument to its maturity rather than invest expecting to sell with the sale proceeds being determined by future interest rates.

QUESTION 5

(a)
Current market prices:

	X-ray	**Yorkie**	**Zulu**
Par value	100	100	100
Maturity	1	3	5
Coupon	4%	4%	8%
Yield	5.0%	5.5%	6.0%
Annuity factor	0.952381	2.697933	4.212364
Coupon strip	3.809524	10.79173	33.69891
Redemption	95.2381	85.16137	74.72582
Value	**99.04762**	**95.9531**	**108.4247**

(b)
The curves B and C have value of 100 (par) when yield is 4% and therefore their coupon rate must be 4% so they are X-ray and Yorkie. Of the two, curve B is steeper and therefore more likely to be the 3yr bond rather than the 1 yr bond because the price is more sensitive to yield changes. This can be confirmed by value at 0% yield – value should be 100 plus 3yrs at 4% = 112 – so B is Yorkie therefore C is X-ray.

Finally curve A must be Zulu – value of 100 at 8% yield and 100+5x8% = 140 at 0% yield. Also it is the steepest sloping curve.

(c)
The real issue for the points where the curves cross each other is the trade off between more cash received earlier (higher coupons) versus the time delay before receiving the biggest cash flow – the return of principal.
Where curves meet the vertical axis is the point, in each case, where the cost of money is zero and the value of the bond is the sum of all cash flows (as in (a) above).

Curves B and C intersect where their values are equal – both are trading at par at a yield of 4%. Curves A and C intersect where, again values are the same at close to 9% yield. Here the small loss in value against par for the short bond (C) at 6% yield over its coupon rate just equates to the loss in value of bond A being only 2% yield over its coupon rate – the steeper decline in value due to its longer maturity.

Curve A and B intersect around 17% yield – again steeper loss in value for the longer bond (9% over coupon rate) equating to a shallower loss in value of the shorter bond (13% yield over its coupon rate).

CALCULATIONS: A and C
At 9.1%:

	C		A
	Xray		**Zulu**
Par value	100		100
Maturity	1		5
Coupon	4%		8%
Yield	9.1%		9.1%
Annuity factor	0.91659		3.87958
Value: Coupon strip	3.666361		31.03664
Value: Redemption	91.65903		64.69582
Value	**95.32539**		**95.73246**

Values very similar, so actual intersection must be close to 9.1%

(Here any calculation base – 9%, 9.5% would make the point adequately)

CALCULATIONS: A and B

		B	A
		Yorkie	**Zulu**
Par value		100	100
Maturity		3	5
Coupon		4%	8%
Yield		17.0%	17.0%
Annuity factor		2.209585	3.199346
Value: Coupon strip		8.83834	25.59477
Value: Redemption		62.43706	45.61112
Value		**71.2754**	**71.20588**

Values very similar, so actual intersection must be close to 17%

(Here any calculation base – 16% or 17% would make the point adequately)

QUESTION 6

Step 1: Calculate ZCRs

	Forward Rates		Zero Coupon Rates	
	FCU	EUR	FCU	EUR
Year 1	2.00%	3.00%	2.00%	3.00%
Year 2	2.50%	3.50%	=((1+2.00%)*(1+2.50%))^0.5 - 1 =2.2497%	= (1.0300*1.03500)^.5 – 1 = 3.2497%
Year 3	3.00%	4.00%	=(1.0200*1.0250*1.0300)^0.333 – 1 =2.4992%	(1.0300*1.0350*1.0400)^0.333 – 1 =3.4992%

Step 2: Value each leg in home currency

	EUR receipt	
Yr		Present value at ZCR
1	430,000	= 430,000 × (1+3.00%) ^-1 = 417,475.73
2	430,000	=430,000 × (1+3.2497%)^-2 = 403,358.19
3	10,430,000	=10,430,000 × (1+3.4992%)^-3 = 9,407,481.97
	Sum PV	**EUR 10,228,315.89**

FCU Payment		
Yr		Present value at ZCR
1	19,200,000.00	=19,200,000 × (1+2.00%)^-1 = 18,823,529.41
2	19,200,000.00	=19,200,000 × (1+2.2497%)^-2 = 18,364,418.94
3	619,200,000.00	=619,200,000 × ((1+2.4992%)^-3 = 575,002,437.63
	Sum PV	**FCU 612,190,385.98**
In EUR that is **(at current FX rate)**		**EUR 7,652,379.82**

So net value is Asset EUR receipt	EUR 10,228,315.89
Liability FCU Pay	EUR 7,652,379.82
Net Value	**= EUR 2,575,936.07 positive value for your company receiving EUR**

(b)
The key issue is the change in FX rate.
A subsidiary issue is the change in interest rates – fixed rates being paid and received have clearly changed substantially as par rates do not correspond to the rates quoted.

When the swap was entered into the market value must have been close to zero. Therefore the exchange rate corresponded to 'close to' EUR10 = FCU 600, i.e. a rate of 60 rather than 80 today.
If that rate still applied the FCU cash stream would be worth

FCU 612,190,386 / 60 = EUR 10,203,173. In this case the swap would be valued at: EUR 10,228,316 – 10,203,173 = EUR 25,143, a much smaller figure. This smaller figure is due to the change in interest rates since the start of the swap.

So – total change in value of the swap is EUR 2,575,936.

Change in value due to exchange rate changes is:
EUR 10,203,173 – EUR 7,652,380 = 2,550,793.

Change in value due to other factors – interest rates is:
EUR 2,575,936 – 2,550,793 = 25,143.

Better scripts stated the direction of influence of each factor.

QUESTION 7

(a)

Uptick probability = $(e^{5\%*0.25} - 0.8) / (1.2 - 0.8) = 0.5314$

Potential outcomes for underlying: 35 × 1.2 = 42 or 35 × 0.8 = 28

Only the lower outcome, 28, has value for the put option – a value of (40 – 28) = 12 with probability of 1-0.5314=0.4686

Put value = probability × value × discount factor
= $0.4686 * 12 * e^{-0.25*5\%}$ = USD 5.55

Put Value = USD 5.55

(b)

Potential outcomes 35 × 1.2 × 1.2 = 50.4, or 35 × 1.2 × 0.8 = 33.6, or 35 × 0.8 × 0.8 = 22.4 i.e. 50.4 or 33.6 or 22.4.

Only the latter 2 give value for the put option:
For 33.6, put future value is (40 – 33.6 = 6.4) × probability (2 × 0.5314 × 0.4686 = 0.4980) = 3.19

For 22.4 put future value is (40 – 22.4 = 17.6) × probability (0.4686 × 0.4686 = 0.2195) = 3.86
Total FV = 3.19 + 3.86 =7.05

PV = FV × $e^{-2\times0.25*5\%}$ = $7.05 \times e^{-2\times0.25*5\%}$ = USD 6.88

Put Value = USD 6.88

(c)

Put call parity formula is
$S0 + P - C = Xe^{-rT}$
$S0 + P - Xe^{-rT} = C$

$35 + 5.55 - 40 \times e^{5\% * 0.25} = C$ = USD 1.05

The call option is out of the money (35 underlying vs strike of 40) with a short time left in relation to the volatility – so it has a low probability of expiring in the money. Probability of jumping to 42 is around half – giving a gain of 2 over strike. Half chance (actually 0.5314) of getting USD 2 gives an expected value of USD 1 – so USD 1.05 looks reasonable.

QUESTION 8

(a)

At 0% of Asset A it is clear that the portfolio represents only the characteristic of Asset B, i.e. risk of 16%, as measured by the standard deviation of returns, the normal measure. At 100% of Asset A it is clear that the portfolio represents only the characteristic of Asset A i.e. risk of 7%.

Taking 25%, 50% and 75% of Asset A, and knowing that there is no connection between returns i.e. that correlation (ρ) is zero, we can calculate the portfolio risk as follows using the formula:

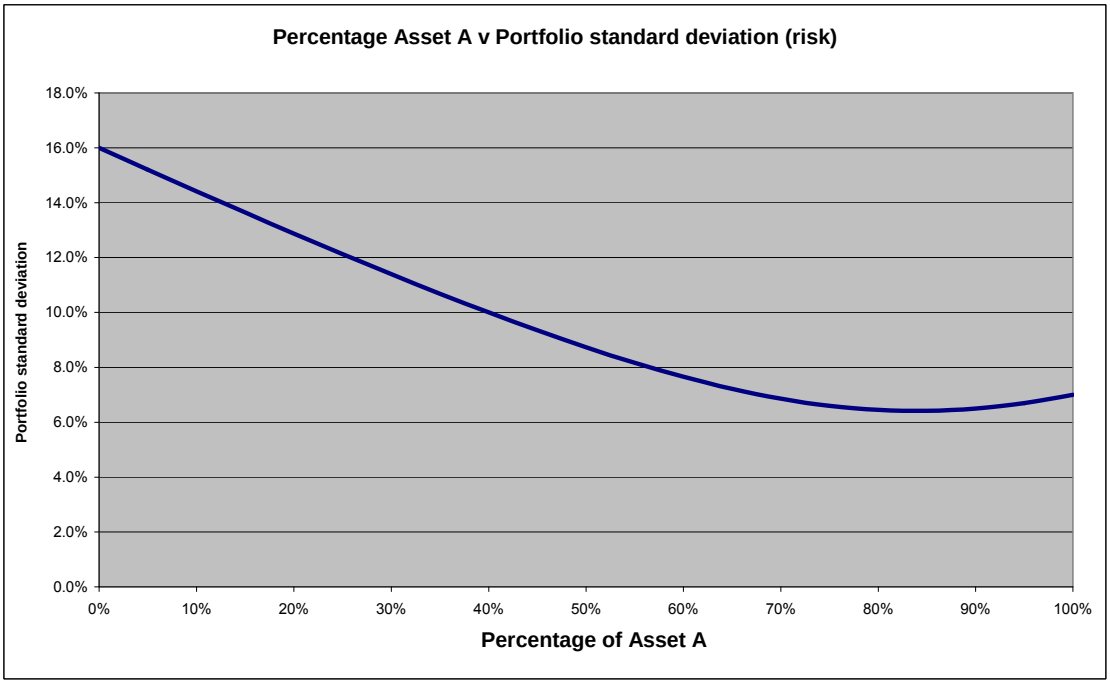
portfolio risk= $\left(a^2 \times \sigma_a{}^2 + b^2 \times \sigma_b{}^2 + 2 \times \rho \times a \times \sigma_a \times b \times \sigma_b\right)^{(½)}$

The table below can be calculated:

% Asset A	Portfolio Risk
0%	16%
25%	12.1%
50%	8.7%
75%	6.6%
100%	7.0%

Note: Any points could be calculated – in fact only one central point would make the point.

Using this table as a base, the sketch below can be drawn



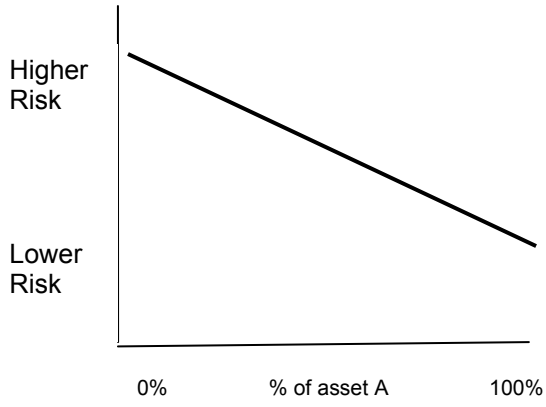
Starting from the right, with a portfolio consisting of 100% of Asset A, i.e. 0% of Asset B, it may seem surprising that the addition of a small proportion of a riskier asset should reduce the overall portfolio risk. The reason for this is that the returns of the two assets are not at all correlated (correlation = 0).

If there were perfect positive correlation between the returns of the two assets then the curve would be a straight line between the two extremes of 100% Asset A to 0% Asset A.

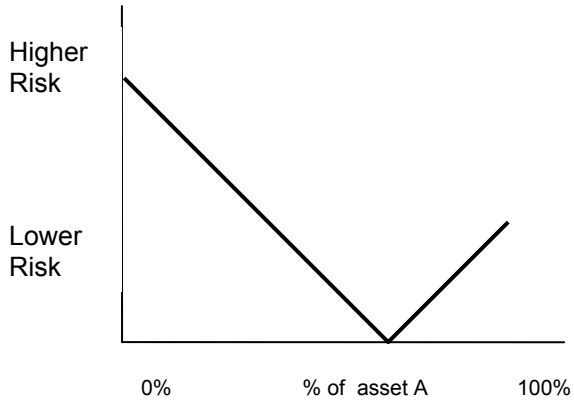
If the returns were perfectly negatively correlated then the curve would comprise two straight lines, one going from the point showing 100% of one asset going to the zero risk axis and the other from there to the point showing 100% of the other asset. This is because correlation of +1 removes any benefit of diversification. Correlation of -1 always provides a mix of assets where the diversification factor exactly offsets the inherent risk of each asset, giving a net zero risk for the portfolio.

From the diagram above it is clear that the correlation between the two assets is between +1 and -1.

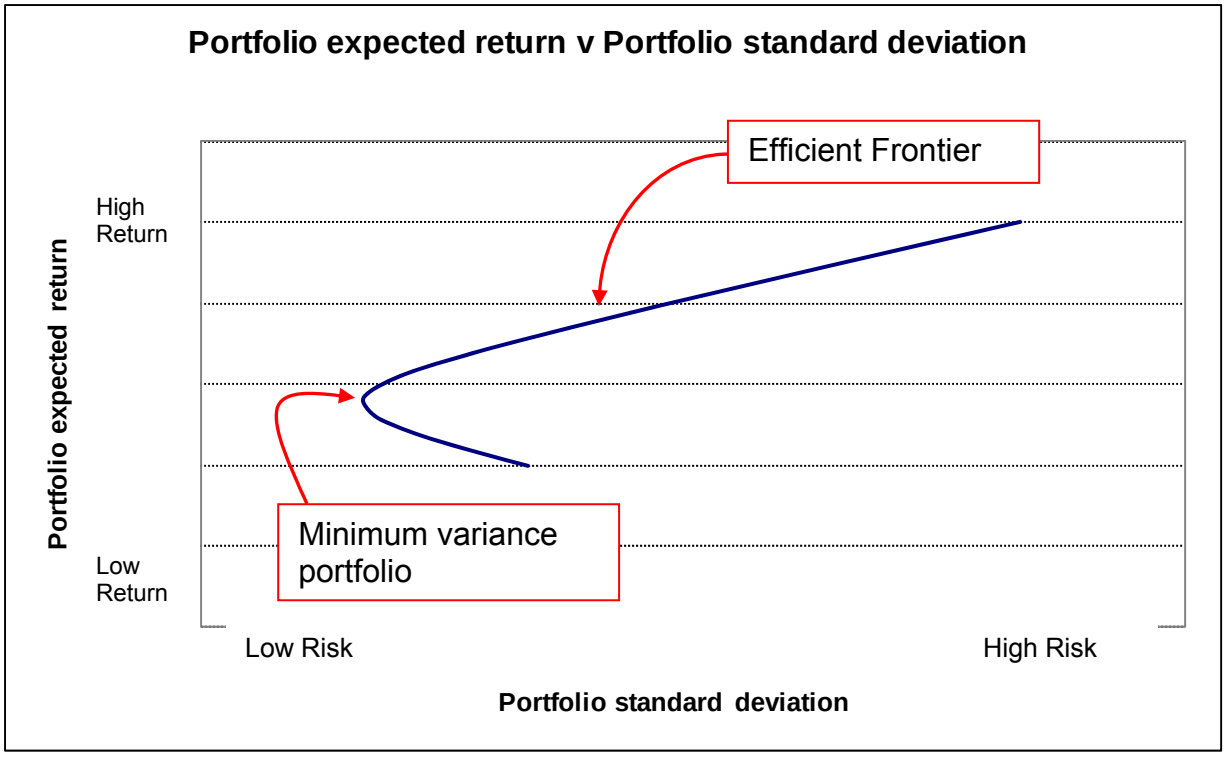
Perfect +ve correlation



Perfect -ve correlation



(b)
The diagram expected is the one below.



From part (a) we know that there is a low risk low return asset – one end of the line – and a higher risk higher return end of the line. So the diagram should follow. Key points to be described:

- The diagram shows the efficient frontier: most return for most risk accepted, or least risk for lower expected return.
- If there were perfect positive correlation between the assets the line would be a straight line.
- If perfect negative correlation the line would go to a point on the zero risk axis.
- So major benefit to diversification is not a gain in return it is a reduction in risk.
- The “nose” is called the minimum variance portfolio (mvp).
- Only the top, above the mvp is efficient, points below are dominated by points above giving higher return for same risk.

QUESTION 9

(a)

Daily VaR (95% confidence) is given by exposure x standard deviation x 1.65, so:

For CHF: 150m x 0.7% x 1.65 = CHF 1,732,500 = **GBP 787,500**

For USD: 300m x 0.4% x 1.65 = USD 1,980,000 = **GBP 1,277,419**

(b)

99% confidence so now need to use 2.33 standard deviations.
Additionally, to change from a 1-day to a 10-day volatility we must multiply by $(10/1)^2$

For CHF: 150m x 0.7% x 2.33 x 10^0.5 = CHF 7,736,512 = **GBP 516,596**

For USD: 300m x 0.4% x 2.33 x 10^0.5 = USD 8,841,728 = **GBP 704,341**

To combine into a portfolio,

VaR (portfolio) = $(VaRA^2 + VaRB^2 + (2 \times correlation \times VaRA \times VaRB))^{0.5}$

= $(3{,}516{,}596^2 + 5{,}704{,}341^2 + (2 \times 0.45 \times 3{,}516{,}596 \times 5{,}704{,}341))^{0.5}$

= GBP 7,934,723

In the exam, these figures could have been rounded more.

(c)

Key assumptions relate to:

- the measurement and consistency of volatility;
    - should measurement use long averages or short – accepting the difficulty of finding any extended period with no trends.
- the measurement and consistency of correlation between the two asset types;
    - again the question regarding use of long time periods or recent short ones – both this and above assume implicitly that the past is representative of the future
- the conformity of the data to normal distributions
    - if leptokurtic … we are relying on tiny part of the distribution, the extreme 1% or 5%, and so 'truth' could be different from our calculated figures.
- the random walk nature of variability to enable the conversion from one-day to ten-day volatilities
    - without this the conversion is meaningless
- No trends over the period of measurement – this is implicit in several of the above points.

All of the above is important to the calculation of VaR, because if any of the conditions is not met then the VaR calculations will be incorrect. The impact of the figures being incorrect is that risk that we are quantifying might be understated – perhaps by a significant degree.

Examiner's Report

Certificate in Financial Mathematics and Modelling October 2011

The Cert FMM exam took place on 5th October 2011. In all 51 candidates sat the exam, up from 49 candidates who sat in April. The pass rate in October was 82.4%. This was particularly pleasing, improving again on the pass rate from April this year which was 65.3%, itself an improvement on the previous sitting.

The paper consisted of nine compulsory questions. It required the application of fundamental principles to exam questions which require an understanding of the course material, rather than just a restatement of that material. All parts of the study material may be examined. So to be confident of passing your exam you should ensure:

1) That you understand the computational techniques illustrated throughout the study materials and can readily re-perform them; and

2) That you can explain clearly the related concepts, principles and relationships, and that you are able to apply them to different situations, including 'joined up' analysis.

The second point appears to offer the most relevant learning for candidates in future exams. Make sure that you can perform the calculations, but also make sure that you can explain what you are doing, why, and what are the assumptions or limitations of the procedure. Candidates who used diagrams typically explained themselves more clearly, more quickly and made fewer errors.

In the exam itself you should ensure that you attempt - so far as possible - all parts of all questions. Any part-questions you do not attempt must necessarily score zero for that part, reducing your prospects of gaining a pass.

The best starting point and reference throughout your studies will be to read and practise the past exam papers which are available for downloading from the ACT's website.

Commentary by question

The paper consisted of 9 compulsory questions ranging between a small 3-mark question and a lengthy option question worth, in total, 22 marks.

QUESTION 1 was conceptually simple. Candidates were required to find the fair price of a set of future cash flows. Those cash flows formed a two stage deferred annuity. Most candidates did well on the question, in fact it was the best answered of all questions on the paper. Those who did not pass typically became confused regarding the timing of cash flows and how to calculate deferred annuity factors. If all else fails in these circumstances, a fairly foolproof method is to start with drawing a timeline and putting cash flows on it.

QUESTION 2 was the shortest question at 3 marks. It required the straightforward calculation of forward rates from given zero coupon rates. Most well-prepared candidates, again, found this straightforward. A few candidates quoted rates that were to a very small number of significant figures: if the question gives rates to 4 decimal places it is advisable to give the answer to 4 decimal places.

QUESTION 3 was the biggest question, with the most marks at stake, 22 marks. It concerned the construction of a bear spread using put options. This question caused problems; it had the second lowest pass rate. The topic is well covered in the material and so the difficulty that it caused was surprising. Graph paper was supplied and candidates were required to draw out the chart of gain/loss against out-turn underlying price, the conventional payoff diagrams at expiry. Some candidates could not attempt this. The question also called for construction of two different bear spread strategies and determination of the breakeven points where the outcome at expiry was the same for both strategies. This part of the question also caused difficulties. Well-prepared candidates did very well on this question.

QUESTION 4 concerned a choice between two forms of borrowing; a discount instrument and a yield instrument. This question was done reasonably well, with a pass rate a little above the average for the paper. Some candidates, though, were still not clear on the difference between these two bases of quotation.

QUESTION 5 gave data for three coupon bonds and a chart showing their values as yields changed. The requirement was first, to say which curve on the chart related to which bond and then to give <u>all</u> the factors that supported the conclusion. Some candidates lost marks because they gave a minimum number of factors, when they might easily have added more – as they showed later in the question. Despite this the question was very well answered.

QUESTION 6 required the valuation of a currency swap. Some candidates ere confused over the means of calculating present value of future cashflows. Both paying and receiving cash flows were fixed, so each cash flow can be valued by using the relevant zero coupon rates. The zero coupon rate had to be derived from the given forward rates. The pass rate was just about average for the paper.

QUESTION 7 was a second option question with 15 marks at stake. This time the question concerned the binomial valuation method. Although some candidates were less than well-prepared, the overall standard was high and the pass rate was above average. The question required a one-step and a two-step valuation. For the candidates who had achieved a clear understanding it presented an opportunity to show their ability.

QUESTION 8 had the lowest pass rate of the paper. It only carried 6 marks, and so those who found it difficult were not penalised too harshly. It concerned the construction of a portfolio from two assets with known risk and return characteristics. Candidates were asked to sketch the shape of the relationship between risk and the mix of assets on a chart with risk as the vertical axis and proportions of one of the assets as the horizontal axis. The intention was to give candidates an opportunity to demonstrate that they understood this relationship and that that the benefit of diversification lies in risk reduction – the lowest risk position is a mix of assets rather than 100% of the low risk asset.

QUESTION 9 concerned Value at Risk. It was reasonably answered although some candidates could have increased their mark noticeably by being aware that most questions concerning Value at Risk are likely to ask about the key assumptions of the technique and the potential for underestimating the risk exposure.

Overall most candidates had prepared well and gave a very good account of themselves to achieve the pass that they deserved.