LEADING TREASURY
PROFESSIONALS

Examination Paper, Solutions and Examiner's Report

Paper:

Certificate in

Financial Mathematics and Modelling

April 2014

QUESTION 1

You are an advisor to Alpha Ltd, a long term investor with a need for income over the next 25 years. You are now investigating two potential investments in new instruments underwritten by a well-known investment bank which may satisfy the income requirements.

Both investments will require an upfront payment now, at Time 0.

No-Growth instrument

Upfront payment	EUR 105 million
Receipts	EUR 15 million per year at the end of Years 5, 6 and 7 and then EUR 10 million per year at the end of Years 8 to 25 inclusive.

Growth instrument

Upfront payment	EUR 115 million
Receipts	Six receipts of EUR 7.5 million at half-yearly intervals, starting after 4.5 years and finishing at end-year 7. Subsequently, there will be a growing series of 36 half-yearly receipts, each 1% greater than the previous receipt commencing with EUR 5 million at 7.5 years and finishing with the end of year 25 payment.

Both instruments are assumed to have the same annual effective market yield of 6%.

Required:

(a) Calculate the net present value of both of the instruments.

(8 marks)

(b) Explain what issues other than NPV are relevant to the decision whether to invest in either instrument.

(2 marks)

(Total 10 marks)

QUESTION 2

Required:

(a) Explain the relationship between maturity, coupon, duration and modified duration for a conventional bond.

(4 marks)

You have a small holding of bonds denominated in Euro. Two of these are:

Bond A
5.5% annual coupon redemption at par in exactly 3 years.

Bond B
4.5% annual coupon, redemption at par in exactly 4 years.

Required:

(b) Calculate the market value, duration and convexity of each of the bonds, assuming a current yield of 5% EAR.

(6 marks)

(c) Calculate the expected new value for each bond after a change in yield to 5.5% EAR using:

i) Modified duration alone
ii) Modified duration and modified convexity

(4 marks)

(d) Explain the rationale and impact of the convexity adjustment when estimating price changes of a bond portfolio.

(2 marks)

(Total 16 marks)

QUESTION 3

DEF Inc is a manufacturer of parts for high-tech smart phones. Over many years, DEF has used long term fixed price contracts for the purchase of the necessary raw materials, many of which would be subject to commodity price variation if bought on the spot market. However, DEF is now considering constructing an option hedge for its purchases rather than agreeing a new fixed price contract.

The prices shown below are the option premia that DEF has been quoted at today's market price of USD 150/kg.

Strike Price	Put Option premium (USD/ kg)	Call Option Premium (USD/ kg)
USD 140/kg	2.20	13.59
USD 150/ kg	5.83	7.32
USD 160/kg	11.78	3.37

DEF is considering the creation of two hedges using the options quoted, which can be bought or sold at the premia shown.

The first hedge under consideration is to create a fixed price at option expiry at minimum or zero net premium. The second hedge under consideration would create a collar so that, before taking account of any premia, the maximum purchase price would be USD 160/kg and the lowest purchase price would be USD 140/kg.
Ignore the cost of carry of any option premia.

Required:

(a) Explain the significance of the fact that at a strike price of USD 150/kg the put premium and the call premium are close to each other in value.

(4 marks)

(b) Explain how you could create an effective fixed price outcome for your material purchases using the options shown.

(2 marks)

(c) For each of the two hedges under consideration over a range of market outturn prices between USD130/kg and USD 170/kg in steps of USD 10/kg:

i) Produce a table to show the outcome for each component of the hedge.

ii) Using the graph paper supplied, draw an accurate chart showing the hedged material cost achieved using the two hedges described and the no-hedge alternative. Show the outturn underlying price on the horizontal axis and the hedged material cost achieved on the vertical axis.

(10 marks)

(Total 16 marks)

QUESTION 4

You are the Treasury Analyst at the TAB Corporation. TAB has an operational requirement for GBP 1 million on 30th October for 6 months (182 days) and has a written commitment from its bank to lend the sum of exactly GBP 1 million on that date. The interest rate agreed is 6-month GBP Libor plus 2.50%.

Market interest rates have been rising rapidly. In order to avoid the risk of the rate to be paid rising even further you have decided to enter a forward rate agreement with TAB's specialist derivatives bank. The rate of the FRA is 3.50%.

In line with derivatives policy, this transaction was reported to the Board. As a result of this report, the Board has now asked for confirmation that the maximum rate that will be paid on this borrowing will be 3.50%.

Required:

(a) Give your response to the Board, explaining why or why not the maximum rate is 3.50%.

(2 marks)

The FRA was entered into on 30 July, 3 months (92 days) before the expected drawdown of the loan. On entering the FRA 3-month GBP Libor was 2.75%.

Required:

(b) Explain how the FRA rate of 3.50% would have been determined, with reference to the 3-month and 9-month GBP Libor rates, and deduce the 9-month GBP Libor rate at the time of entering the FRA.

(4 marks)

At 30 October the outturn 6-month GBP Libor fixing is 4.35%

(c) Calculate the all-in effective cost of the loan, as an EAR, on the assumption that TAB must borrow exactly GBP 1 million regardless of any FRA settlement payment.

(5 marks)

(Total 11 marks)

QUESTION 5

Gaggle, an Irish company reports in EUR. Some time ago Gaggle entered a long term USD liability which was hedged with a currency swap to convert the net liability to EUR. That swap arrangement now has exactly 4 years to maturity and the counterparty has raised the possibility of a credit support annex. The proposal is that if the value of the swap exceeds USD 10million to either party then the party to whom the swap is a liability should provide cash collateral, in USD, equal to the value of the swap to support the credit worthiness of the arrangement.

This cash collateral would remain in an escrow account to the value of the swap until the swap value had fallen below the threshold figure of USD 10million when it could be withdrawn.

The terms of the swap are as follows.

Maturity	4 years
EUR leg	fixed 3% annual payment on EUR 100million principal
USD leg	fixed 2.5% annual receipt on USD 110million principal

Re-exchange of principal at maturity.

Current market data:

Interest rate data

Period	EUR zero coupon rate	USD zero coupon rate
1 year	3.80%	3.20%
2 years	3.90%	3.25%
3 years	4.00%	3.30%
4 years	4.10%	3.35%

Exchange rate EUR 1 = USD 1.3339

Required:

(a) Calculate the current value of the swap to Gaggle in its reporting currency given today's market information above.

(6 marks)

(b) Quantify separately the impact on the current swap value of:

i) A parallel shift in the USD yield curve increasing by 0.5% while the EUR curve remains the same

ii) A change in the exchange rate to EUR 1 = USD 1.2000

(4 marks)

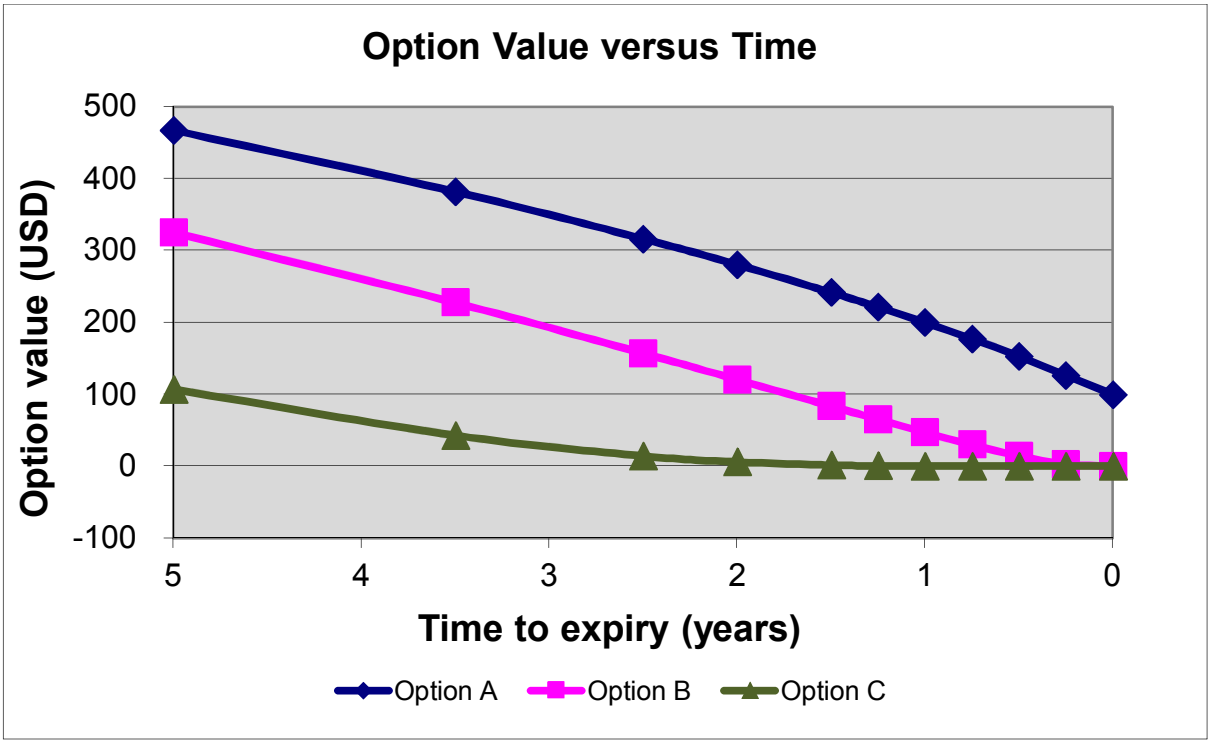
c) Comment on the impact for Gaggle of introducing the credit support annex.

(3 marks)

(Total 13 marks)

QUESTION 6

The chart shows the value of three options as their time to expiry decays. All three options are over the same underlying asset with the same expiry date. Note that the horizontal axis shows time reducing towards the right of the chart.



Required:

Explain as fully as you can the main features of the chart, differentiating between the three options.

In your explanation state whether each option is a call or a put option and explain why the relationship between option value and time appears different for each.

(10 marks)

QUESTION 7

Your firm, Risky PLC, is all equity funded because it operates in a high risk business. Your beta has just been measured at 1.75, confirming the equity market's view of your riskiness.

You are about to start a new project, which comprises an initial phase to try to prove viability for a new business followed by a major expansion if viability can be proved. The initial phase has costs of USD50 million and annual cash inflows of USD12 million per year. A time limit of 5 years has been put on the initial phase of the project, at the end of which it will be either dropped if viability is still unproven, or expanded.

If the expansion is undertaken, then the further investment expected is USD350 million. The current expectation of cash inflows, is a present value of USD250 million, but the present value of revenues is subject to an annual standard deviation of 35%.

Risk free rate	5% EAR.
Equity risk premium	7% p.a.

(a) Given that Risky PLC is all equity funded, what is the net present value of the initial phase of the project?

(6 marks)

(b) Explain the nature of the embedded real options in many projects subject to investment appraisal.

(5 marks)

(c) Calculate the value of the real option to expand that is embedded in the initial project to prove viability.

(6 marks)

(Total 17 marks)

QUESTION 8

You are looking for a market anomaly and have come across the following situation.

GBP Libor quote	
3 months	1.0000%
6 months	1.0000%
9 months	1.0000%
12 months	1.0000%

(a) Calculate the 3 v 6, 6 v 9 and 9 v 12 month forward rates implied by these quoted rates, giving your answers as a percentage to 4 decimal places, as in the question.

(4 marks)

(b) Explain the relationship between the Libor quotes and the forward rates calculated in part (a). Use calculations where you think appropriate.

(3 marks)

(Total 7 marks)

FORMULAE

1. Present value of an annuity; Annuity Factor

$PV = A_1 \times AF_{(r,n)}$

$AF_{(r,n)} = \frac{1}{r} \times [1-(1+r)^{-n}]$

2. Sample variance

$$Var[X] = \frac{1}{n}\sum_{i=1}^{n}(x_i - E[X])^2$$

3. Estimated population variance

$$Var[X] = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - E[X])^2$$

4. Covariance

$$\sigma_{xy} = E[(x-\mu_x)(y-\mu_y)]$$

5. Coefficient of correlation

$$\rho_{xy} = \frac{\sigma_{xy}}{\sigma_x \sigma_y}$$

6. Macaulay Duration

$$\text{Duration (D)} = \frac{\text{Sum (PV} \times \text{t)}}{\text{Sum (PV)}}$$

7. Modified duration

$$D_{MOD} = \frac{\text{Macaulay Duration(D)}}{[1+r]}$$

8. Convexity

$$\frac{\text{Sum}(PV \times t \times (t+1))}{\text{Sum}(PV)}$$

9. Modified convexity

$$C_{MOD} = \frac{\text{Macaulay Convexity}(C)}{[1+r]^2}$$

10. Option pricing: single period binomial model (probability of an uptick)

$$p = \frac{e^{rt} - d}{u - d}$$

11. Option pricing: Black Scholes model

$$C = S_0 \times N(d_1) - X \times e^{-rT} \times N(d_2)$$

$$P = X \times e^{-rT} \times N(-d_2) - S_0 \times N(-d_1)$$

$$d_1 = \frac{\ln\left(S_0 / X\right) + \left(r + \sigma^2 / 2\right) \times T}{\sigma \times \sqrt{T}}$$

$$d_2 = \frac{\ln\left(S_0 / X\right) + \left(r - \sigma^2 / 2\right) \times T}{\sigma \times \sqrt{T}} = d_1 - \sigma \times \sqrt{T}$$

12. Option pricing: put-call parity relationship

$$S_0 + P - C = Xe^{-rT}$$

13. VaR holding period adjustment

$$\sigma_{t2} = \sigma_{t1} \times \sqrt{\frac{t_2}{t_1}}$$

14. Correlated VaR

$$VaR_{AB} = \sqrt{VaR_A^{\,2} + VaR_B^{\,2} + \left(2 \times \rho_{AB} \times VaR_A \times VaR_B\right)}$$

Standardised normal distribution table

Cumulative Distribution Function for the Standard Normal Random Variable [N(x)] where x ≥ 0.

The table shows values of N(x) for x ≥ 0.

The table can be used with interpolation.

For example:

$$N(0.4245) = N(0.42) + 0.45 \times [N(0.43) - N(0.42)] = 0.663 + 0.45 \times (0.666 - 0.663) = 0.664$$

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	0.500	0.504	0.508	0.512	0.516	0.520	0.524	0.528	0.532	0.536
0.1	0.540	0.544	0.548	0.552	0.556	0.560	0.564	0.567	0.571	0.575
0.2	0.579	0.583	0.587	0.591	0.595	0.599	0.603	0.606	0.610	0.614
0.3	0.618	0.622	0.626	0.629	0.633	0.637	0.641	0.644	0.648	0.652
0.4	0.655	0.659	0.663	0.666	0.670	0.674	0.677	0.681	0.684	0.688
0.5	0.691	0.695	0.698	0.702	0.705	0.709	0.712	0.716	0.719	0.722
0.6	0.726	0.729	0.732	0.736	0.739	0.742	0.745	0.749	0.752	0.755
0.7	0.758	0.761	0.764	0.767	0.770	0.773	0.776	0.779	0.782	0.785
0.8	0.788	0.791	0.794	0.797	0.800	0.802	0.805	0.808	0.811	0.813
0.9	0.816	0.819	0.821	0.824	0.826	0.829	0.831	0.834	0.836	0.839
1.0	0.841	0.844	0.846	0.848	0.851	0.853	0.855	0.858	0.860	0.862
1.1	0.864	0.867	0.869	0.871	0.873	0.875	0.877	0.879	0.881	0.883
1.2	0.885	0.887	0.889	0.891	0.893	0.894	0.896	0.898	0.900	0.901
1.3	0.903	0.905	0.907	0.908	0.910	0.911	0.913	0.915	0.916	0.918
1.4	0.919	0.921	0.922	0.924	0.925	0.926	0.928	0.929	0.931	0.932
1.5	0.933	0.934	0.936	0.937	0.938	0.939	0.941	0.942	0.943	0.944
1.6	0.945	0.946	0.947	0.948	0.949	0.951	0.952	0.953	0.954	0.954
1.7	0.955	0.956	0.957	0.958	0.959	0.960	0.961	0.962	0.962	0.963
1.8	0.964	0.965	0.966	0.966	0.967	0.968	0.969	0.969	0.970	0.971
1.9	0.971	0.972	0.973	0.973	0.974	0.974	0.975	0.976	0.976	0.977
2.0	0.977	0.978	0.978	0.979	0.979	0.980	0.980	0.981	0.981	0.982
2.1	0.982	0.983	0.983	0.983	0.984	0.984	0.985	0.985	0.985	0.986
2.2	0.986	0.986	0.987	0.987	0.987	0.988	0.988	0.988	0.989	0.989
2.3	0.989	0.990	0.990	0.990	0.990	0.991	0.991	0.991	0.991	0.992
2.4	0.992	0.992	0.992	0.992	0.993	0.993	0.993	0.993	0.993	0.994
2.5	0.994	0.994	0.994	0.994	0.994	0.995	0.995	0.995	0.995	0.995
2.6	0.995	0.995	0.996	0.996	0.996	0.996	0.996	0.996	0.996	0.996
2.7	0.997	0.997	0.997	0.997	0.997	0.997	0.997	0.997	0.997	0.997
2.8	0.997	0.998	0.998	0.998	0.998	0.998	0.998	0.998	0.998	0.998
2.9	0.998	0.998	0.998	0.998	0.998	0.998	0.998	0.999	0.999	0.999
3.0	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999
3.1	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999
3.2	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999
3.3	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
3.4	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

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HEALTH WARNINGS

Solutions are not totally comprehensive

Answers set out here are not totally comprehensive of all of the relevant points which could be made in a fuller discussion of the selected topics.

Additional valid and relevant points - if clearly set out - will always be credited in your exam, even if they are not incorporated into the published solutions.

Alternative solutions

Any clear route to a valid solution will be awarded full marks. You do not need to use the same calculation methods as those illustrated.

The answers illustrated are not necessarily the only ones possible. For reasons of space, alternative valid assumptions, methods and answers are not normally set out in the published solution, but they are fully credited in the exam.

Slightly different numerical solutions may also be calculated, depending on the rounding of intermediate figures.

Explanatory & illustrative diagrams

You are strongly encouraged to incorporate diagrams within your solutions to explain and illustrate where relevant. For reasons of space, these solutions are not comprehensive of all of the possible relevant diagrams. Candidates who produced relevant diagrams were awarded credit for doing so.

Question 1

(a)

Amounts in EUR

No-Growth Instrument

Annuity Factor years 5-7 inclusive	$=(1/6\%)*(1-(1+6\%)^{-3})$ =	2.673012
Value in Year 4	15 * 2.673012 =	40.09518
PV 5-7	**$=40.09518 * (1+6\%)^{-4}$**	**31.75914**
Annuity Factor years 8 - 25 inclusive	$=(1/6\%)*(1-(1+6\%)^{-18})$ =	10.8276
Value in Year 7	= 10 * 10.8267 =	108.276
PV 8-25	**$=108.276 *(1+6\%)^{-7}$ =**	**72.00975**
Total PV	= 31.75914 + 72.00975 =	103.7689
NPV total	=-105 + 103.7689 **=**	**-1.23112**

Growth Instrument

Semi-annual rate, i.e. per period rate = $(1+6\%)^{0.5}$ -1 = 2.9563%

Annuity	**Initial phase**		
	Annuity Factor	$=(1/ 2.9563\%)*(1-(1+ 2.9563\%)^{-6})$ =	5.425046
	Period 8 Value	= 7.5 * 5.425046 =	40.68785
	PV initial phase	**$=40.68785 * (1+2.9563\%)^{-8}$**	**32.22859**
	Perpetuities - growth phase		
	Perpetuity factor (1/r)	=1/(2.9563% - 1%) =	51.1169
	Period 14 Value	5 * 51.1169 =	255.5843
	PV growth phase	$=255.5843 * (1+2.9563\%)^{-14}$ =	169.9782
	Final perpetuity to be deducted		
	Perpetuity factor (1/r)	=1/(2.9563% - 1%) =	51.1169
	First payment	$= 5*(1+1\%)^{36}$ =	7.1538
	Period 50 Value	=7.1538 * 51.1169 =	365.6821
	PV to be deducted	$365.6821 * (1+2.9563\%)^{-50}$ =	85.2034
	Growing annuity value		
	= Difference in perpetuities	=169.9782 – 85.2034	84.7748
	Total Value of Instrument		
	Sum of PVs	32.22859 + 84.7748 =	**117.0033**
	NPV of the instrument	-115 + 117.0033 =	**2.0033**

Comment (not required by the question):

Essentially the NPV rule – No-growth instrument has a negative NPV so do not invest.

Growth instrument is positive NPV so do invest. Although there would be a qualification about scale of potential gain – need to be sure that risk is correctly judged for 2% gain on investment over a 25 year investment.

(b)
Key issues to be covered from the following:

- Counterparty risk. There are not many 'well-known investment banks' that would justify the 2% NPV gain on a 25-year risk.
- The growing instrument also has a slightly higher risk in the later years due to the growth – assuming that 1% per half year is greater than inflation – so that exacerbates the risk on the counterparty. Effectively this extends the duration of the instrument.
- Liquidity risk – the instruments may not be tradable during their life meaning that exit may not be possible.
- The calculations assume a constant effective market yield throughout the life of the instruments; values may change
- The size of the investment may rule out their applicability – the funds may not be readily available.

Question 2

(a)

Maturity is the life of the bond's cashflows, the time until all cashflows cease – normally with a redemption payment.

Coupon is the recurring periodic payment throughout the life.

Duration is the PV-weighted average of the cashflows' timings, so that if all cashflows were replaced by one single cashflow, then the timing of that cashflow would be the duration of the bond. Another way of putting it would be that the zero coupon bond (a single cashflow) that varies in market value with respect to yield would have a maturity equal to this bond's duration.

When coupon is zero, maturity = duration – because there is a single cash flow and maturity and duration must be the same for a single cashflow.

As coupon increases, value is delivered earlier so duration reduces while maturity remains the same. In the extreme case, for a perpetuity – just coupons with no redemption payment, duration would be the PV-weighted average of coupons and maturity would be infinite.

Modified duration is always lower than duration because duration is always divided by (1+r) (assuming r is positive). As r increases modified duration reduces proportionately more below duration. As r approaches zero, modified duration approaches duration.

(b)

BOND A

Year	Flow	DF	PV	PVT	PVT(T+1)
1	5.5	0.9524	5.2381	5.2381	10.4762
2	5.5	0.9070	4.9887	9.9773	29.9320
3	105.5	0.8638	91.1349	273.4046	1093.6184
			101.3616	288.6200	1134.0266

DURATION	**=288.62/101.3616**	**2.8474**
CONVEXITY	**=1134.0266/101.3616**	**11.1879**

BOND B

Year	Flow	DF	PV	PVT	PVT(T+1)
1	4.5	0.9524	4.2857	4.2857	8.5714
2	4.5	0.9070	4.0816	8.1633	24.4898
3	4.5	0.8638	3.8873	11.6618	46.6472
4	104.5	0.8227	85.9724	343.8896	1719.4482
			98.2270	368.0004	1799.1566

DURATION	**=368.0004/98.227**	**3.7464**
CONVEXITY	**=1799.1566/98.227**	**18.3163**

(c)

Using the formula **– *Modified Duration***

BOND A Duration estimate
=101.3616 – (101.3616 *
(2.8474/(1+5.0%))*0.5%) **= 99.9872**

Duration + Convexity estimate
=99.9872 +
$0.5*(101.3616*(11.1879/(1+5\%)^2)*\ 0.5\%^2$ **= 100.0001**

Check: 5.5% coupon bond must trade at par on 5.5% yield

BOND B Duration estimate
=98.2270 – (98.2270 *
(3.7464/(1+5.0%))*0.5%) = **96.4746**

Duration + Convexity estimate
$=96.4746 + 0.5*(98.2270*(18.3163/(1+5\%)^2)*$
$0.5\%^2)$ **= 96.4950**

(d)

The 'duration alone' estimate must underestimate the new value of an instrument for a rise or a fall in the yield. This is because the modified duration is a straight line estimate and, because the actual price versus yield curve is steeper at low yields than at high yields, the straight line tangent to the curve is always equal to or below the curve – never above.

The convexity adjustment takes account of this degree of 'curviness' by always adding to the duration estimate for rises and falls in yield.

Question 3

(a)

The two premia are close to each other because the strike price of USD 150 is close to the theoretical forward price. The two premia would be exactly equal, ignoring bid offer spreads, if the strike price was exactly equal to the forward price – (put call parity when market value now (S_0) = PV of strike price). When this is the case (the two premia exactly equal) then the two options combine to produce the same result as a forward contract – if the premia were not equal there would be an arbitrage opportunity between the two option premia combined and the forward contract – they deliver the same thing at different prices.

This is put call parity – and can be expressed mathematically as

$S_0+P-C = X\ e^{-rt}$

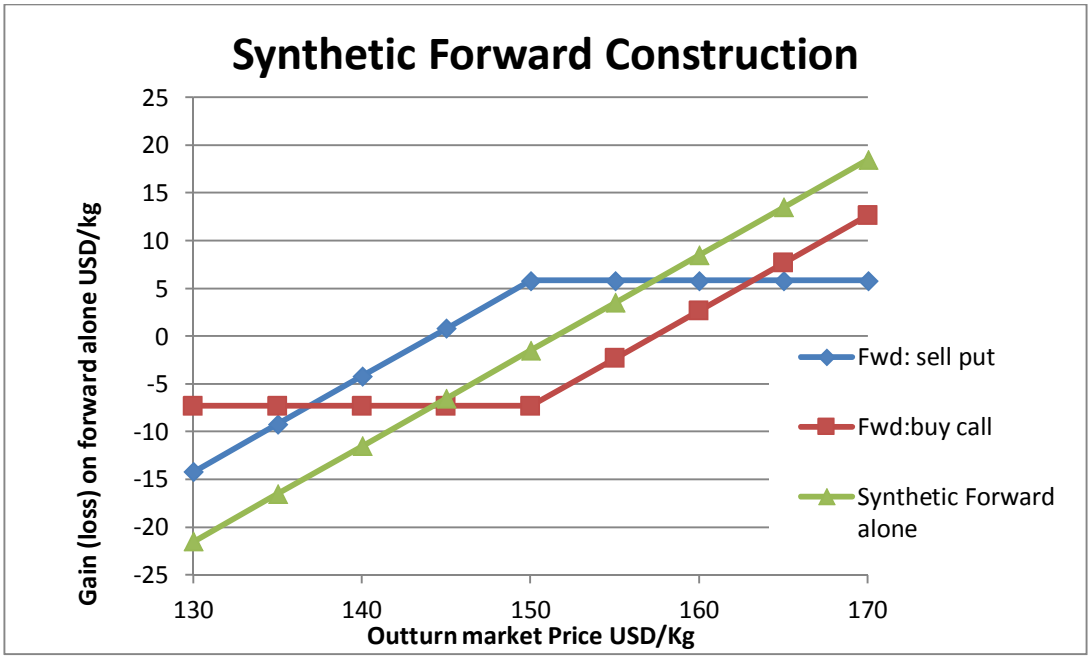
where $X\ e^{-rt}$ is the PV of the strike price and S_0 is the current market price. When the two are equal they cancel out so that P must equal C.

(b)

The fixed price outcome can be achieved by selling a put and buying a call at a strike price of USD 150/kg. Cost (net premium) will be USD 7.32 less USD 5.83 = USD 1.49/kg

This will create a synthetic forward contract. Any strike price could be chosen to give a constant outcome – at the cost of a net premium, but the change in net premium will be offset by the gain/loss on the options, leaving the fixed future price very similar. (Identical in a perfect market).

The net premium will be zero if the market price is equal to the present value of the strike price – put call parity – when the put premium received equals the call premium paid.



[Sketch not required as part of the answer]

This is a synthetic forward that will fix the effective cost of the material at USD 1.49 above the strike price of the two options i.e. USD 151.49/kg.

(c)

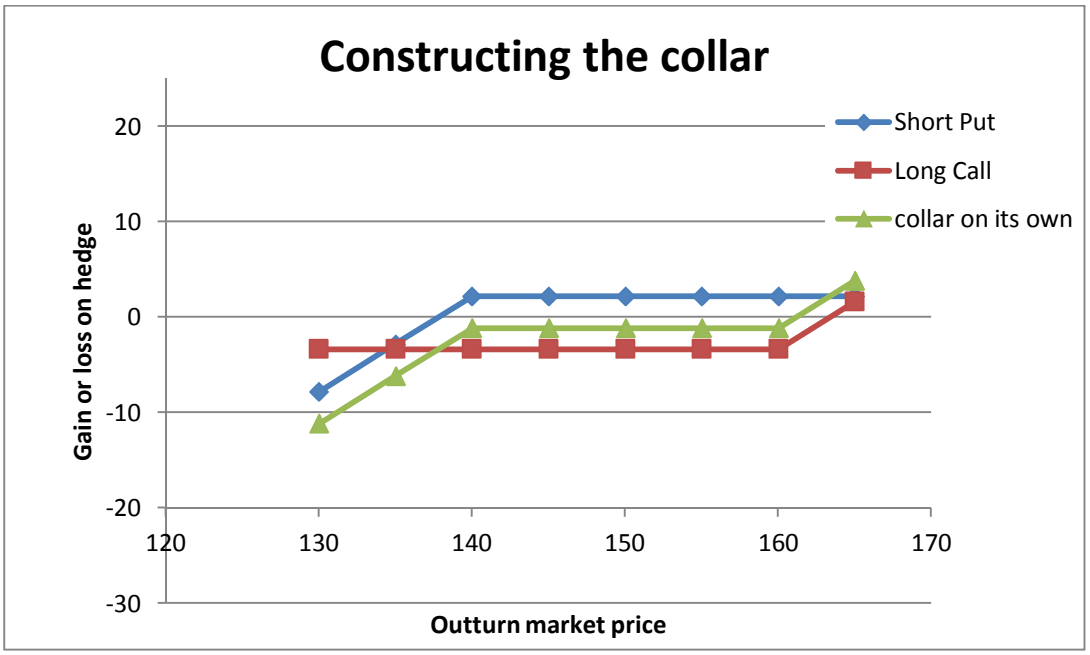
Table – Collar

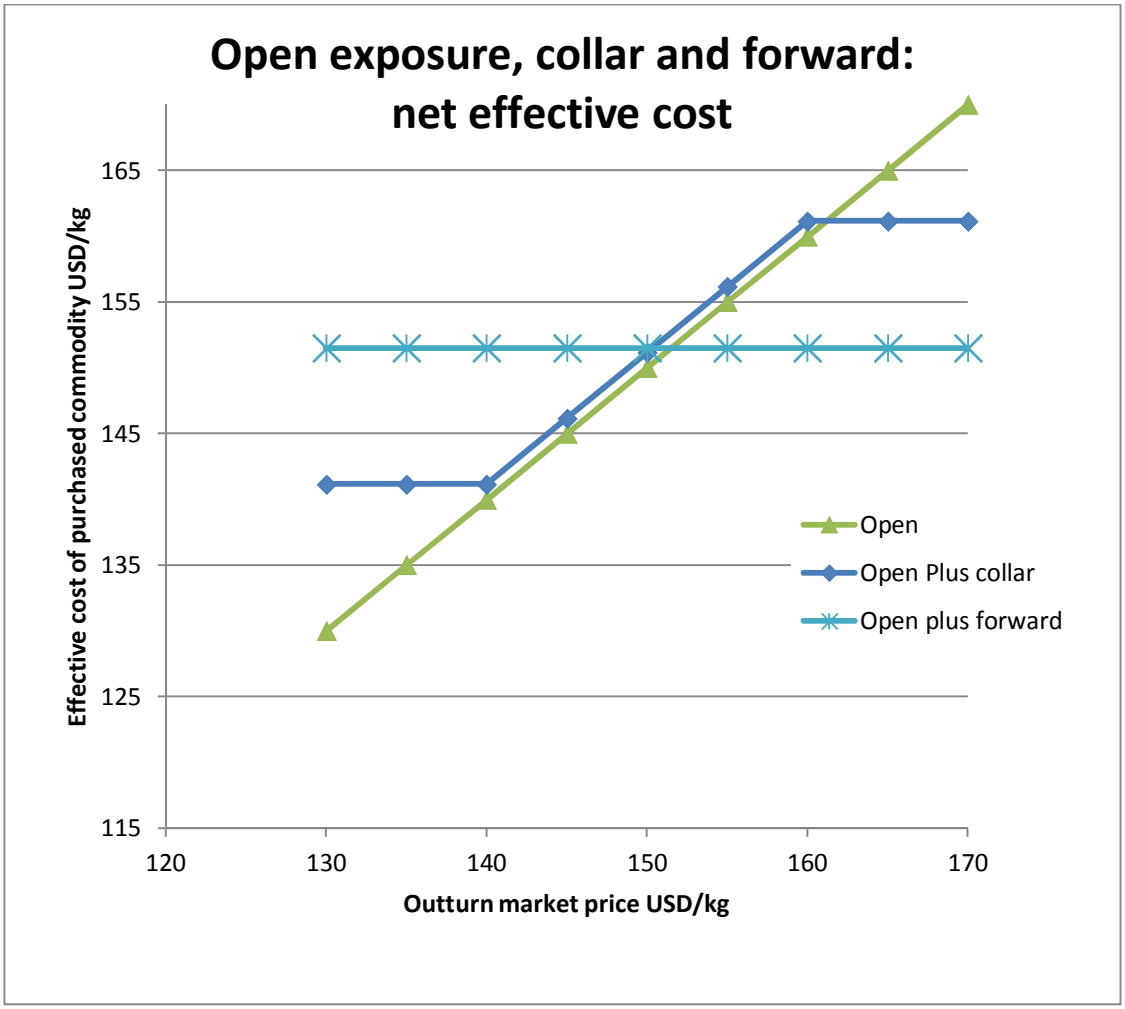
Open Exposure USD/kg	Collar components (income/ (cost)) Short Put USD/kg (X=140)	Long Call USD/kg (X=160)	Collar impact on ***purchases*** [(income)/cost] = - Short put - long call USD/kg	Open + Collar As cost Open Plus collar USD/kg
130	-7.8	-3.37	11.17	141.17
140	2.2	-3.37	1.17	141.17
150	2.2	-3.37	1.17	151.17
160	2.2	-3.37	1.17	161.17
170	2.2	6.63	-8.83	161.17

Table – Forward fix

Open Exposure UDS/kg	Forward components (income/(cost)) Sell put USD/kg (X=150)	Buy call USD/kg (X=150)	Forward net As **cost** [(income)/cost] = -put - call USD/kg	Open + forward As **cost** **USD/kg**
130	-14.17	-7.32	21.49	151.49
140	-4.17	-7.32	11.49	151.49
150	5.83	-7.32	1.49	151.49
160	5.83	2.68	-8.51	151.49
170	5.83	12.68	-18.51	151.49

The chart showing the construction of the collar below and the chart showing the synthetic forward construction shown previously were not required in the exam but are included to show the build up of the final chart, which was required.





Question 4

(a)

The FRA rate is the 'guaranteed' level of the benchmark NOT the rate that the company will pay. Adding the credit spread, the company will actually pay very close to 3.5% Libor plus 2.5% credit spread = 6.0% in total.

(b)

The FRA rate is effectively the forward forward rate for a Libor borrowing arranged now to start in 3 months time. If an investor/borrower can invest for 3 months at 2.75% and sequentially invest for a further 6 months at 3.5%, then the total return for that construction must be the same as for a straight 9 month investment – otherwise there would be an arbitrage opportunity. Usually the FRA rate would be calculated from 3month and 9 month rates – here we must work backwards – both routes must yield the same result.

3 month Libor	2.75%
3 - 9 month forward rate i.e. FRA rate	3.50%

Invest GBP 100

3 months (92 days) time	GBP 100.6932
6 more months (182 days later)	GBP 102.4505
9 months Libor (274 days)	=((102.4505/100)-1)*365/274 **= 3.264%**

Check: Forward (FRA) rate higher for rising yield curve, looks right.

(c)

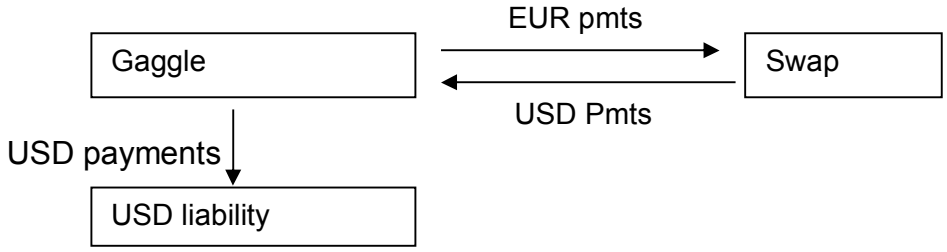
	GBP
Amount	1,000,000
Outturn rate	4.35%
FRA rate	3.50%

Rate difference	0.85%
Settlement payment *if paid at end*	=rate difference * amount * 182 days = GBP 4,238.36
PV of settlement payment at start	GBP 4,148.38 ←**receipt by TAB**

		Loan GBP	**Inflow/(Outflow) GBP**	**Total GBP**
Cash flows	start	1,000,000	4,148.38	1,004,148.38
	end	1,000,000	((4.35%+2.5%) *GBP1m * (182/365)) = (34,156.16)	1,034,156.16
	Cost of borrowing		=1,034,156.16 – 1,004,148.38	=30,007.79
	periodic borrowing cost		=30,007.79/1,004,148.38	=0.029884
	EAR		= (1+.029884)^365/182 -1	**= 6.0832%**

= FRA rate + margin (3.5%+2.5% = 6%) plus 0.0832% EAR

Question 5



		USDm	**EURm**
Coupon	2.5%*110	= 2.75	3.0%*100 = 3.00
Principal		110	100
Zero coupon rates			
Year:			
1		3.2000%	3.8000%
2		3.2500%	3.9000%
3		3.3000%	4.0000%
4		3.3500%	4.1000%
Discount factors			
Year:			
1		0.9690	0.9634
2		0.9380	0.9263
3		0.9072	0.8890
4		0.8765	0.8515

PV calculations: **USD Swap Receipts**

T	USDm Flow	DF	PV	EURm equivalent
1	2.75	0.9690	2.66	
2	2.75	0.9380	2.58	
3	2.75	0.9072	2.49	
4	112.75	0.8765	98.83	
			106.57	÷ 1.3339 = **79.89**

EUR Payments

T	EURm Flow	DF	PV	
1	3.00	0.9634	2.89	
2	3.00	0.9263	2.78	
3	3.00	0.8890	2.67	
4	103.00	0.8515	87.71	
Sum			96.04	96.04

Difference = NPV = swap value **- 16.15**

(b) i) Yield curve shift for USD

PV calculations: **USD Swap Receipts**

	USDm			Total EURm
		Revised		
T	Flow	DF	PV	equivalent
1	2.75	0.9643	2.65	
2	2.75	0.9290	2.55	
3	2.75	0.8941	2.46	
4	112.75	0.8598	96.94	
Sum			104.6	**78.42**

EUR Payments

	EURm			
T	Flow	DF	PV	
1	3.00	0.9634	2.89	
2	3.00	0.9263	2.78	
3	3.00	0.8890	2.67	
4	103.00	0.8515	87.71	
Sum			96.04	96.04

Difference = NPV **-17.62**

Increase in liability of 17.62 – 16.15 = EUR1.47m

(b) ii) Exchange rate shift

PV calculations: **USD Swap Receipts**

	USD			Total EUR
T	Flow	DF	PV	equivalent
1	2.75	0.9690	2.66	
2	2.75	0.9380	2.58	
3	2.75	0.9072	2.49	
4	112.75	0.8765	98.83	
Sum			106.6	÷1.2000 = 88.80

EUR Payments

	EUR			
T	Flow	DF	PV	
1	3.00	0.9634	2.89	
2	3.00	0.9263	2.78	
3	3.00	0.8890	2.67	
4	103.00	0.8515	87.71	
Sum			96.04	96.04

Difference = NPV **-7.24**

Reduces liability by 16.15 – 7.24 = EUR 8.91m

(c)

There are two broad issues here; the impact on the cashflow exposure and the counterparty risk.

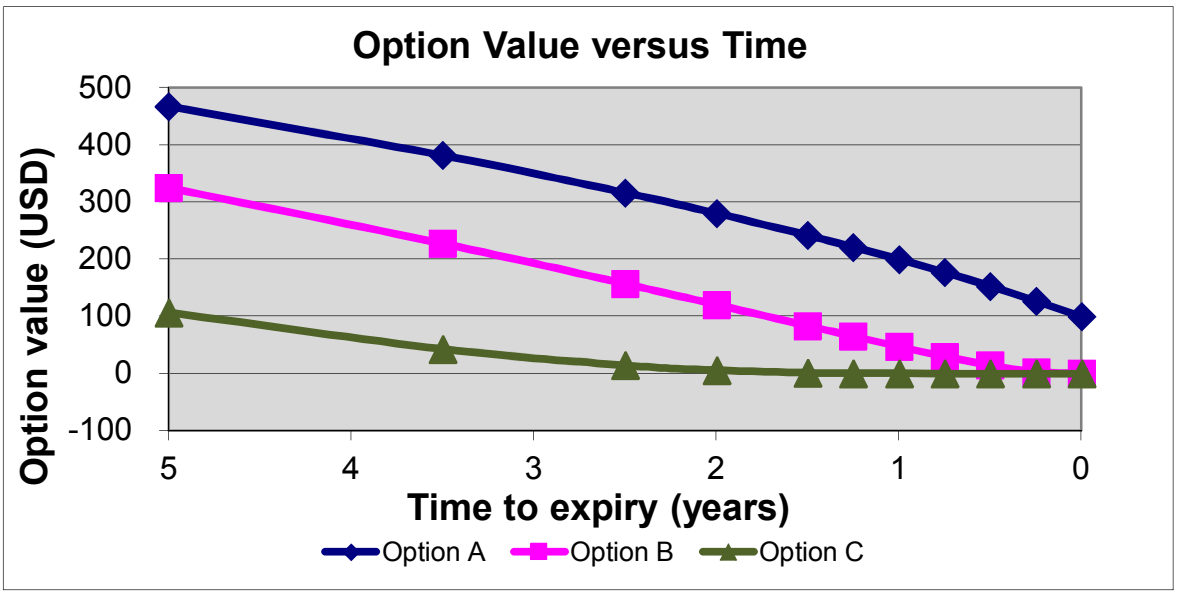
The impact of the credit support annex is that whenever the swap value exceeds USD 10m, the company is required to pay USD 10m, or more depending on the swap value, into an escrow account. If interest rates change or if exchange rates change then the amount might increase or decrease until the net swap value falls below USD10m again, when the payment is returned. Any collateral remaining at the end of the swap is returned.

The point of the swap is to avoid the risk of cashflows and values changing. Without the swap the capital value of the USD liability might have changed in EUR terms or the EUR value of USD payments might have changed. The purpose of the swap was to prevent either of these impacting on the cashflows of the company.

The effect of the credit support annex is to re-introduce this variability. A change in rates (interest rates or exchange rates) results in sudden calls for unpredicted payment. Part of the purpose of the swap is therefore negated to leave Gaggle back with the cash flow exposure they wished to avoid.

There is also a credit risk on the swap counterparty and this risk is addressed by the CSA. Since the financial crisis of 2007/8 the counterparty risk cannot be ignored. So the CSA limits the counterparty risk to a maximum of USD10m, but at the cost of a key reason for the swap.

Question 6



All three options are call options because they clearly increase in value with time to expiry. Put options with a strike above current market value would increase in value with time over very low values for time but then reduce in value as time increases beyond about one year.

The reason for the increase in value of call options as time to expiry increases is that asset values are assumed to 'drift' upwards at the risk free rate: if the underlying asset price is expected to increase, that is good news for call options – a longer call option will have a higher value – and bad news for put options – high asset prices mean lower payouts for put option holders. With 5 years remaining until expiry there is plenty of time for the underlying asset to increase in value, but with 6 months remaining to expiry there is little scope for price growth. In this chart, all options increase in value between 2 and 5 years remaining so must be call options.

Option A is a call with strike 100 below current market (i.e. market value exceeds strike) – if expiry was today it would be worth 100. Because expiry may not be today, as time increases and given the assumed updrift in asset values, plus the impact of volatility, the option is worth more. This is the effect of the combination of volatility, the risk-free rate and time remaining which aggregate to 'time value'.

Option B has a strike such that it is out of the money, but only just, as shown by the fall to zero value at around 6 months or less. This could be caused by either a low volatility and a strike only just below market price or a higher volatility and a strike rather more below current market value.

Option C is so far out of the money that even 2 years of updrift and volatility don't create any value, although if there is more time remaining there is value in the option.

Question 7

(a)

Annual cash flow	USD 12m	
Annuity		
Rf	5%	
Years	5	
beta	1.75	
Equity Risk Premium	7%	
Ke	**=5% + 1.75 * 7%**	**=17.25%**
Annuity Factor 5 yrs	$=(1/17.3\%)*(1-(1+17.3\%)^{-5})$	3.1810
S_0 = Annual cash * AF	=12m * 3.1810 = **USD 38.1726m**	
NPV of the project	= -50 + 38.1726 = **USD -11.8274m**	

(b)

There are three options embedded in almost every corporate project that is the subject of investment appraisal. Those three embedded options are:

i. the option to delay the project
ii. the option to expand the project
iii. the option to abandon the project

The *option to delay* is only potentially of value if the firm has exclusive rights over a project. This ensures that the project remains intact. Without exclusive rights another competitor could start the project and gain 'first mover advantage', effectively leaving the firm at a disadvantage. In financial terms it is very like deciding whether to exercise an American option that is in the money, or wait hoping that the option will move more into the money. For the option to delay revenues may improve or costs may fall making the delay may be beneficial. This is the volatility of the project cashflows. This option is a call option.

The *option to expand* is present on most projects – start small to keep costs and risks relatively low and then, if all goes well, scale up. Some projects preclude the possibility – HS2 to Birmingham for example, but even that is a small step towards a bigger project to expand to Manchester, Leeds and Scotland. The option can only be retained if the potential for expansion is built in; for example ensuring that the initial investment can cope with further demands. This option is also a call option.

The *option to abandon* is a put option – it becomes valuable at low market values, in other words when outcomes are less than hoped for. The abandonment is normally a possibility if there is a fallback position so that if expectations are not met there is an alternative course of action that generates more than just 'walking away'.

(c)

This is an option to expand and can be valued using Black Scholes formula. Strike is the USD 350 million investment and S_0 is the expected PV of the cashflows, USD 250 million.

Risk free rate is quoted as 5% EAR, so r_{cc} = ln(1+5%) = 4.879%

Amounts in USD millions

Underlying price now = **So**	250
Strike price = **X**	350
Risk free rate per annum = **r**	4.879%
Standard deviation per annum = **σ**	35%
Time to maturity (years) = **T**	5
ln(So/X)	-0.336
$r + \sigma^2/2$	*0.110*
$(r + \sigma^2/2) \times T$	0.550
$\sigma \times T^{0.5}$	0.783
$d_1 = [\ln(So/X) + (r+ \sigma^2/2) \times T]/[\sigma \times T^{0.5}]$	0.273
$d_2 = d_1 - \sigma \times T^{0.5}$	-0.510
$N(d_1)$	0.608
$N(d_2)$	0.305
Call value calculation:	
$So \times N(d_1)$	151.902
Discount Factor: e^{-rT}	0.784
Present value of strike price: $X * e^{-rT}$	274.234
$X * e^{-rT} * N(d_2)$	83.693
Call value $C = So * N(d_1) - X * e^{-rT} * N(d_2)$	**68.21**

Call Value is the value of the option to expand – **USD 68.21m**.

This means that the total NPV for the initial project is USD -11.83m + USD 68.21 = USD 56.38.

Question 8

(a)

period	Libor (ZCR)	Periodic rate	Periodic forward rate	**forward quote** = period rate * 4
0-3 mths	1%	0.2500%	= 0.2500%	**1.0000%**
0-6 mths	1%	0.5000%	= (1+.50%)/(1+.25%) -1 =0.2494%	**0.9975%**
0-9 mths	1%	0.7500%	= (1+.75%)/(1+.50%) -1 =0.2488%	**0.9950%**
0-12 mths	1%	1%	= (1+.1.00%)/(1+.75%) -1 =0.2481%	**0.9926%**

(b)

This relationship appears at first to contradict expectations, but in fact it does not. Forward rates appear to be declining while ZCR (Libor) is flat in apparent contradiction of theory. However, it does not give any contradiction, the issue is that Libor is calculated as a simple interest rate, but because the forward rates are quoted as quarterly rates, the compounding effect inherent in a quarterly quote makes it appear that the forward rate is lower than the zero coupon rate.

To demonstrate:

Converting both to EAR:

ZCR Libor	3 month	EAR = $(1+.2500\%)^4 -1$	= 1.0038%
	6 month	EAR = $(1+.5000\%)^2 -1$	= 1.0025%
	9 month	EAR = $(1+0.7500\%)^{(12/9)} -1$	= 1.0012%
	12 moth	EAR = $(1+1\%)^{\wedge 1} -1$	= 1.0000%
FWD rates	0-3 month	EAR = $(1+1.0000\%/4)^4 -1$	= 1.0038%
	3-6 month	EAR = $(1+.9975\%/4)^4 -1$	= 1.0012%
	6-9 month	EAR = $(1+.9950\%/4)^4 -1$	= 0.9987%
	9-12 month	EAR = $(1+.9926\%/4)^4 -1$	= 0.9963%

This shows that, when quoted on the same basis, the ZCR rate (Libor) is falling and, as expected, the forward rate is falling faster.

April 2014 FMM Examiners Report

The Cert FMM exam took place on Tuesday 8th April. The total number of candidates for the exam was 56, a substantial increase over the October 2013 exam when 38 people sat the exam. 34 candidates passed giving a pass rate of 60.7%, a very small percentage increase over the 60.5% achieved at the previous sitting. The pass rate has been similar for the last three sittings so that it is worth reiterating that there is no fixed pass rate. The pass mark is 50% and any candidate who achieves that level of performance will pass.

The paper consisted of 8 compulsory questions ranging between a 7-mark on yield curve analysis and explanation and a lengthier 17-mark question on corporate valuation and real options. The paper required the application of fundamental principles to exam questions which require an understanding of the course material, rather than just a restatement of that material. All parts of the study material may be examined. So to be confident of passing your exam you should ensure:

(1) That you understand the computational techniques illustrated throughout the study materials and can readily re-perform them and, where required, apply the concept rather than a formula to the exam question.

(2) That you can explain clearly the related concepts, principles and relationships, and that you are able to apply them to different situations, including 'joined up' analysis.

(3) Whenever a formula is used, be aware of the limitations of the formula and think whether it is valid in the circumstances of the question.

The first point raises the issue of understanding; the exam tries to test whether a candidate really understands what a calculation is doing rather than just applying a formula. The second point emphasises the importance of making sure that you can explain what you are doing, why, and what are the assumptions or limitations of the procedure. Candidates who used diagrams typically explained themselves more clearly, more quickly and made fewer errors. The third point becomes important when non-standard situations are being discussed.

In the exam itself you should ensure that you attempt - so far as possible - all parts of all questions. Any part-questions you do not attempt must necessarily score zero for that part, reducing your prospects of gaining a pass. In a few cases, there was a suspicion that candidates had skip-read the question, missing the final phrase '… and comment on your result.'

The best starting point and reference throughout your studies will be to read and practise the past exam papers which are available for downloading from the ACT's website. However, do not overlook the fact that new material is introduced from time to time and so new exam topics do arise.

Commentary by Question

Question 1

This question was the valuation of deferred and growing annuities and perpetuities. On the whole it was well answered, although some candidates were not well prepared. Surprisingly many candidates gave little attention to part (b) of the question, given that it carried 20% of the question's marks. This should be a key learning point for all candidates – attempt all parts of the question seriously.

Question 2

This was a question about bond values and their risk measures; duration and convexity. Many candidates scored highly here but a minority could not explain the relationship between maturity, coupon, duration and modified duration. For the well-prepared this was not a difficult question, as evidenced by the candidates who scored full marks.

Question 3

This 16-mark question concerned the hedging of a commodity purchase using options. The requirement was to create a synthetic forward contract and a collar from the selected few options for which data was available. Similar questions have proved difficult in the past, but this was answered well by most candidates. This is an important topic for practical treasury.

Question 4

Question 4 was a relatively straightforward FRA question. Surprisingly a significant minority of candidates found it difficult and many mistakes were made. It was one of the questions where the average mark was low. Understanding the detail of how returns are calculated to determine the effective cost of borrowing is an important component of the course and so it can be expected to arise frequently in this exam.

Question 5

This concerned a currency swap. A value for the swap was required given interest rate and exchange rate data; superimposed on the value calculation was a credit support annex with terms described. Most candidates could cope well with the swap valuation and even the variation in interest rates and exchange rates. But some were left struggling to think of the implications of a CSA that required depositing funds when the swap value moved adversely. This may be the phenomenon mentioned above, that candidates sometimes rush on to the next calculation rather than complete the 'comment' section of the solution.

Question 6

This 10-mark question showed a chart of option values against time and asked for an interpretation of what was shown. This requires thinking from first principles. This sort of question has been asked before but some candidates still seemed to be unprepared. The best candidates could cope but others struggled, the average mark for this question was one of the lowest of the paper.

Question 7

This question required a project valuation and the application of real option theory. This is the first time that real options have been covered in the exam although the material has been in the syllabus for some diets. Well-prepared candidates coped well while others had clearly not covered material that had not appeared in the exam before. This question also had a low average mark.

Question 8

Very surprisingly this last question received a low pass rate. Surprising because it is a question that has been asked several times before in different guises. It relates to a situation where zero coupon rates (Libor) appear to be flat but the periodic forward rates calculated appear to be falling. In fact the illusion of rates falling arises from the fact that the periodic quarterly rates are implicitly compounded and zero coupon rates quoted as simple rates. When both are quoted as EARs they are both seen to be falling slightly, in line with expectations.

Overall this is a very pleasing result with an acceptable pass rate. Well-prepared candidates coped well, but less well-prepared candidates appeared to struggle.