A quantitative approach

Executive summary

- The members of well-funded pension schemes adopting an liability-driven investment strategy presumably benefit from the increased certainty of receiving their pensions, but does the sponsor? And, what about the Pension Protection Fund, both now and when it changes its levy calculation rules? Financial models exist for answering these questions but are not well known. The second part of this feature looks at stochastic deflator models.

Just to recap, the first part of this article, in the June issue of *The Treasurer*, discussed how a very simple contingent claims model could be used to investigate the impact of various investment and other strategies on the relative wealth shared by shareholders and pension scheme members. These models can provide useful insights into the directions in which wealth transfer might occur, but great caution should be exercised in interpreting quantitative results. This concluding article considers a more sophisticated type of model that can be used to produce a quantitative wealth transfer analysis in which users can have a fair degree of confidence.

WHY STOCHASTIC DEFLATOR MODELS? In a world of uncertain outcomes, the benefits of stochastic models are now fairly well appreciated, so the answer to this part of our question will be taken as a given. Deflators are powerful modelling tools that can be used to put a market-consistent value on complex and uncertain future cashflows. Although creating economic models that generate deflators as well as traditional economic outputs can be difficult, once built, their use to value cashflows simulated using the economic outputs of the model can be quite simple.

The glossary on page 34 explains some of the concepts in more detail, but, from the point of view of the treasurer, understanding the typical output of such models (as long as they have been carefully constructed by people who know what they are doing) is probably more important than understanding the maths. As Andrew Smith said of a different, but no easier, approach to optimisation in his 1996 *British Actuarial Journal* paper, *How Actuaries Can Use Financial Economics*: "The mathematics is an entertaining mix between collective risk theory, modern portfolio theory and geometry." For most of us, Smith's use of the word 'entertaining' in this context is akin to the use of the word 'interesting' in the ancient Chinese curse: "May you live in interesting times."

The advantages of deflator models include:

- the placing of economic values on the stakes of all stakeholders;
- consistency of these values with market conditions, assuming the deflator model is calibrated against relevant market indicators;
- the alternative risk-neutral approach (see glossary), which involves changing the measure of the underlying distribution so that the risk premium is zero, seems counterintuitive; and
- they do not rely on the existence of a perfect dynamic hedge, which is necessary, for example, in option pricing models such as Black-Scholes.

A SIMPLE EXAMPLE Let us consider conceptually how a simple contingent claims model using deflators over a single time period would be built taking into account only three single-class stakeholder groups: company shareholders, company debt holders (effectively ignored in the simple contingent claims model described last month) and scheme members. Any surplus generated is shared between shareholders and scheme members.

Before considering the impact of different scenarios (here we shall consider investment strategy, but we could also have looked at, say, contribution strategy), we need to agree some common assumptions, which will include the return distributions for the company's assets as well as for the pension scheme's bonds and equities (including the equity risk premium): see the table below. Successive sets of projections

Variable	Case One	Case Two
Equity beta	1.0	2.5
Company-specific risk	10%	30%
Scheme liabilities/market cap	50%	100%
Company debt/market cap	25%	50%
IAS 19 funding ratio	100%	100%

JOHN HAWKINS AND **TIM KEOGH** CONCLUDE THEIR TWO-PART SERIES ON PENSION SCHEME DE-RISKING WITH A DISCUSSION OF STOCHASTIC DEFLATOR MODELS.

could also, of course, assume different values for some or all of these. We shall also consider two cases, where the risk of the sponsor firm is rather different.

We shall discuss what the results might look like from a conventional (deterministic) discounted cashflow analysis, a traditional asset liability model and a stochastic deflator model.

Cashflow analysis The first thing to note is that the firm's risk characteristics have no impact on the analysis. With the scheme starting on a fully funded (albeit not particularly strong) basis, there is a positive probability of surplus being generated if a risky (high-equity) investment strategy is followed; this would not be the case if a hedged bond strategy was adopted. As long as the pension scheme members share in this surplus, a risky investment strategy might make some sense. This is illustrated in *Figure 1*, where the equity strategy always delivers a better result, with no uncertainty.

Asset liability model analysis Again, firm risk has no impact, but a consideration of the output of a stochastic asset liability model soon makes the investment risk clear. At the end of the period there will be a range of outcomes, but as long as an equity risk premium is incorporated the median outcome will still show a surplus and trustees may still be tempted to adopt a high-risk investment strategy. *Figure 2* shows a typical outcome distribution for 100% debt and 100% equity strategies, with the +/- 1 (68%) and +/- 2 (95%) standard deviation outcomes highlighted for the latter. As long as trustees believe that any deficit can and will be made good by the sponsor, targeting a surplus in which they will share can still seem attractive.

Deflator model The critical factor missed by a traditional asset liability model is the sponsor insolvency risk, but this is picked up automatically in a properly constructed contingent claims deflator model that will automatically discount high-risk cashflows at a higher rate. We now need to consider each of the cases in turn.

CASE 1 For our purposes this is a risk-less company. By moving from a debt to an equity strategy, members may receive additional benefits, so that their share of the firm's value is increased very slightly at the expense of shareholders (with bondholders almost unaffected). This is illustrated in *Figures 3* and *4*.

Figure 1: Growth in liabilities and assets: deterministic

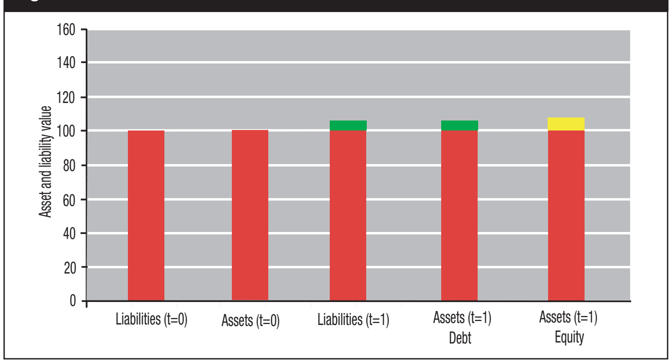

Figure 2: Growth in liabilities and assets: stochastic

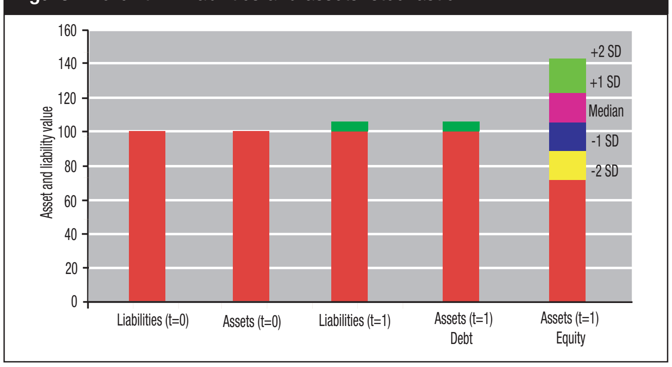

Figure 3: Investment strategy and value transfer: Case 1

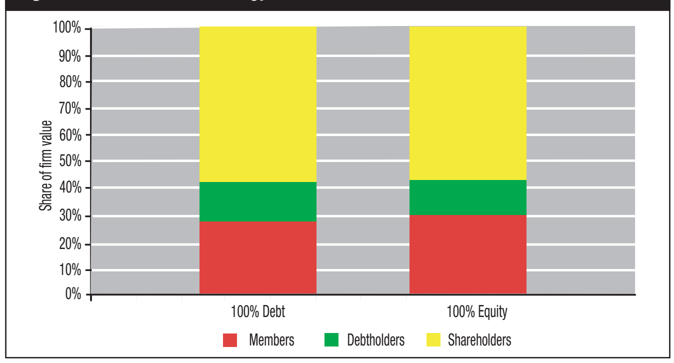

Figure 4: Change in stakeholder values: Case 1

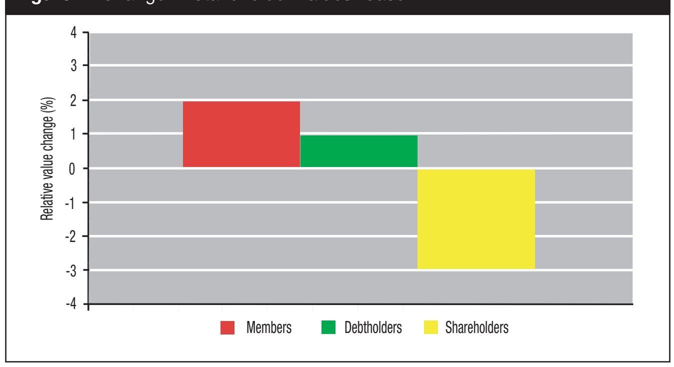

Figure 5: Investment strategy and value transfer: Case 2

Figure 6: Change in stakeholder values: Case 2

CASE 2 We are now dealing with a company with high specific risk, combined with relatively high levels of both debt and pension scheme liabilities in relation to its market capitalisation. Moving from a debt to an equity strategy would result in a significant transfer of value from scheme members to shareholders, with a slight reduction in the value attributed to bondholders, as shown in *Figures 5* and *6*.

This very simple model should give the general idea of how the deflator modelling technique can be applied, although it is a non-trivial task to extend the model to multiple time periods and to incorporate other variables, such as interest rates, inflation and longevity. In principle, it is also possible to take into account dynamic investment and scheme contribution strategies.

However, as with the basic contingent claims model, one of the most important extensions is to incorporate other stakeholders, such as the Pension Protection Fund (PPF). Incorporating the PPF within the model can be critical in a number of scenarios, as might be concluded from considering the following.

In its current form, the PPF levy is not directly influenced by the pension scheme's asset allocation. So all other things being equal, increasing investment risk is likely to result in value being transferred from the PPF to the sponsor. At funding levels below PPF liabilities, increased funding is likely to result in value transfer from the sponsor to the PPF (net of any tax benefit). This is sometimes referred to as the dead zone.

STOCHASTIC DEFLATOR MODELS IN PRACTICE For the sponsor or trustee board of a pension scheme seriously interested in pursuing the type of analysis described here, the first question will be whether to build their own model from first principles or customise an existing proprietary model. Proprietary models are nearly always designed in such a way as to obviate known problems, such as the debt pricing issue referred to last month. Also, when the time available for analysis is constrained, it can be spent on fine-tuning (for example, in regard to key company-specific stakeholders) and proper calibration, rather than basic model building.

STAKEHOLDERS Although this article is directed at company treasurers, it ought to be clear that the consequences of actions taken to de-risk pension schemes should be of interest to a wide range of stakeholders and that a quantitative approach is helpful (if not mandatory) to understand them anywhere near fully. Not only this, but the results of such actions can sometimes be counterintuitive.

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GLOSSARY

ASSET LIABILITY MODEL A stochastic financial model for projecting pension scheme assets and liabilities under a range of economic scenarios, thereby providing a methodical framework for the identification and quantification of investment risk and the evaluation and comparison of alternative strategies.

CONTINGENT CLAIM Any asset whose value depends on some other asset. Options provide a good example, but there are many others.

DETERMINISTIC MODEL A mathematical model in which the parameters and variables are not subject to random fluctuations, so that the system is at any time entirely defined by the initial conditions chosen.

MARKET-CONSISTENT In relation to the valuation of, say, pension scheme liabilities, a value that would be consistent with the market price of a portfolio of assets exactly hedging the liabilities.

MERTON MODEL A model proposed by Robert Merton for assessing the credit risk of a company by characterising its equity as a call option on its assets.

MODEL CALIBRATION Changing the values of input parameters in an attempt to match observable outputs, such as the price of marketable securities.

PUT-CALL PARITY The relationship between a European-style put option and a European-style call option on the same underlying asset with the same exercise price and maturity. Put-call parity states that the pay-out profile of a portfolio containing an asset plus a put option is identical to that of a portfolio containing a call option of the same strike on that same asset (with the rest of the money earning the risk-free rate of return).

RISK-NEUTRAL Term used to describe an investor who values risk at a constant value. As an investor trait, risk-neutral lies between risk-aversion and risk-seeking: a risk-neutral investor will accept exactly the same interest rate (usually the risk-free rate) for all assets. The value that a risk-neutral investor assigns to a financial instrument is usually different from the expected value of the financial instrument based on market prices. Since the latter will be affected by the price the market is willing to pay for risk, actual market prices will vary from risk-neutral prices and risk-neutral probabilities will vary from actual probabilities.

RISK-NEUTRAL PROBABILITY Term used to refer to probabilities which, when used as weights in an expected-value calculation, will reproduce the market value of financial instruments. In general, risk-neutral probabilities differ from real-world probabilities because the market does not assign value in the same way that a risk-neutral individual would. They do, however, provide an important analytical approach.

STOCHASTIC DEFLATOR Conventional stochastic models of cashflows – for example of a pension scheme – produce a range of outcomes that are then discounted at a single discount rate estimated outside of the model. However, it is possible (and computationally convenient) to develop models in such a way that a discount rate can also be generated in relation to each outcome; the term deflator has been coined to describe this stochastic discount rate.

STOCHASTIC MODEL A mathematical model that takes into consideration the presence of randomness in one or more of its parameters or variables, so that the predictions of the model do not give a single point estimate, but a probability distribution of possible estimates.