Spotlight Quiz
Yield Curves

Question 1
Match the cashflow patterns below (A to C) to the appropriate yield (X to Z).

A a single cash flow occurring in the future and being valued now
B a single cash flow occurring at period n in the future being valued at period n-1
C a finite series of cash flows; one small ‘coupon’ payment at the end of each period and a single redemption payment with the final ‘coupon’ payment, all being valued now.

X the par yield
Y the zero coupon yield
Z the forward yield

(a) A is represented by X, B represented by Y, C represented by Z
(b) A is represented by Y, B represented by Z, C represented by X
(c) A is represented by Z, B represented by X, C represented by Y
(d) A is represented by Z, B represented by Y, C represented by X
(e) not a clue!

Answer
The right answer is (b) A is represented by *Y, B* represented by *Z, C* represented by *X*

The zero coupon, or spot, yield relates to a single future payment in the future being valued now (time zero). It must not be confused with the par yield which is the single yield appropriate for valuing a cash flow pattern very like a redeemable coupon bond. In years past the par yield was referred to as the gross redemption yield. The forward yield relates to a single future cash flow being valued one period before its occurrence (usually) NOT now.

Question 2
Once one of either the par, zero coupon or forward yield curve is known we can use underlying financial logic to determine the other two. Which one of the following is NOT used in this determination?

(a) arbitrage
(b) weighted average
(c) bootstrapping
(d) backwardation
(e) not a clue!

Answer
The right answer is (d) backwardation

The most likely underlying principle to be used is that of arbitrage – that alternative routes to the same end must yield the same result. If this were not true then arbitrageurs could use alternative routes to create cashflows different to those quoted in the market and make a riskless profit (or loss if they are not very good arbitrageurs!). But the zero coupon rate is, in effect, the weighted average of the component forward rates, as can be demonstrated by calculating the amount that would result from an initial investment now followed by continually re-investing at the forward rate. This gives the total that must also be the result of investing at the zero coupon rate – but the calculation is effectively that of a *weighted average. Similarly the par rate is the weighted average of the series of zero coupon rates for each coupon cash flow and the redemption payment weighted by their relative amounts. Bootstrapping is not really an 'underlying financial logic' but it is a method of determining the zero coupon rate from the relevant par rate. Backwardation is the situation where it is cheaper to buy a commodity in the future than it is to buy it now. With no income associated with holding the commodity, the forward price is normally higher to reflect the holding cost.*

Question 3
Imagine that the term structure of interest rates (the academically accurate term for what we all call the yield curve) shows rates rising as maturity (or more strictly, tenor) increases i.e. a normal yield curve.

Under these circumstances, which order to the curves appear?

	The Highest (above the others)	In the Middle	The Lowest (below the others)
(a)	forward	par	zero coupon
(b)	par	forward	zero coupon
(c)	zero coupon	forward	par
(d)	forward	zero coupon	par
(e)	par	zero coupon	forward
(f)	zero coupon	par	forward

(g) it varies depending on a complex interaction of offsetting factors
(h) not a clue!

Answer
The right answer is (d) forward, zero coupon, par

For a tenor of 1, a single period, all rates must be the same – they represent the same set of cash flows. If we extend to 2 periods, then the sum invested at the T_{0-1} forward rate could be rolled forward into the T_{1-2} forward rate. The zero coupon rate for T2 must represent the same T0 investment and the same T2 return – so the amount resulting at T2 must be the same whichever route is taken. As an example if T_{0-1} is 5% and T_{1-2} is 6% then £100 invested would yield £105 after one period and just over £111 (£111.30)at T2 (£105 × 1.06%). If the zero coupon rate is to yield the same then the zero coupon rate must be around 5.5%. In fact it comes out to be 5.4988% - very close to the average of 5% and 6%.

Question 4
Normally the zero coupon and par curves are fairly close together. Which of the following factors would cause the par curve to be closest to the zero coupon curve?

(a) a steep upward sloping curve
(b) a steep downward sloping curve
(c) a low prevailing rate environment
(d) a high prevailing rate environment
(e) none of these makes any difference
(f) not a clue!

Answer
The right answer is (c) a low prevailing rate environment.

The par rate for any given maturity is given by the weighted average zero coupon rate for each of the cashflows of the par bond. In a low interest rate environment the coupon of the par bond will be low, *therefore the weighting will move more in favour of the zero coupon rate of the redemption amount – i.e. the zero coupon rate.*

Question 5
Some treasurers believe that financial maths is a step too far in sophistication. If you know that the forward rates for the next two years are as follows, can you estimate what would be a good 2 year swap price?

Forward rate T_{0-1} = 3.25%
Forward rate T_{1-2} = 4.65%

(a) 3.934%
(b) 3.944%
(c) 3.954%
(d) 3.964%
(e) any of these will do!

Answer
The right answer is (a) 3.934%

First we need the zero coupon rates. For the first period the ZCR is the same as the forward, 3.25%. For the second period, the ZCR is given by $[(1+3.25\%)\times(1+4.65\%)]^{(1/2)} - 1 = 3.9476\%$
There is a formula for calculating the par rate (or swap rate) from the relevant zero coupon rates. It is

$$\frac{1 - \text{Discount Factor}_n}{\text{Cumulative DF}_n}$$

It refers to the discount factor derived from the zero coupon rates and the sum of all of the discount factors up to year n, again using zero coupon rates. In this case the formula becomes:

$$\frac{1-1.039476^{-2}}{(1.0325^{-1}+1.039476^{-2})} = \frac{0.07451218}{1.8940} = 0.039341 \text{ or } 3.9341\%$$

The answer can be proved by using 3.9341% as the coupon on the par bond, i.e. the bond trading at par and showing that, when discounted at the zero coupon rate, the PV is 100.0000, or par.
.

T	***Flow***	***DF***	***PV***
1	*3.9341*	*0.968523*	*3.8102663*
2	*103.9341*	*0.925488*	*96.189744*
			100.0000

Question 6
Which is the best way to describe the difference between yield curves?

(a) they are different curves observed in the market
(b) one curve is observed and the others are calculated from that
(c) there is one underlying yield curve, the differences are all to do with increasing apparent complexity of interacting factors
(d) there is one underlying yield curve, the differences are all to do with quoting conventions and the assumed pattern of cash flows
(e) it’s all too mathematical for me, I’m interested in strategy!

Answer

The right answer is (d) there is one underlying yield curve, the differences are all to do with quoting conventions and the assumed pattern of cash flows.

Each curve is based on a rate quote which assumes a different set of cashflows; therefore each quote will have a different figure to reflect the different assumed cashflows set within the same underlying situation.