Spotlight Quiz - Portfolio Theory and Risk
Worked Solutions

Question 1
In combining assets with different risk / return characteristics, which of the following generates the advantage that the portfolio can provide over investments in the individual assets?

(a) enhanced return from the portfolio over the individual assets
(b) reduced risk from the portfolio relative to the individual assets
(c) both enhanced return and reduced risk from the portfolio over the individual assets
(d) there is no advantage for the portfolio over the individual assets

Answer
The right answer is (b) reduced risk from the portfolio over the individual assets.

The combined return is always determined by the weighted average of the returns of the portfolio components, so there is no enhancement of portfolio return. When combining risk though, there is a reduction in portfolio risk relative to the weighted average of the component assets unless the correlation between the assets is +1, i.e. perfect correlation. Most people would not expect much of a portfolio effect under these circumstances – imagine owning one BP share and being told that you needed more shares in your portfolio .. so you buy another BP share! This would be an example of perfect correlation. In practice most assets have a less than perfect correlation.

Question 2
One of the assumptions upon which portfolio theory is based is that investors are rational, although many practitioners find this assumption hard to concede. Which of the following gives the usual definition of a rational investor?

(a) prefers a higher return for a given risk and prefers a lower risk for a given return
(b) invests in passive funds rather than active funds
(c) invests only in fully diversified portfolios
(d) hasn’t invested in equities since 2000

Answer
The right answer is (a) prefers a higher return for a given risk and prefers a lower risk for a given return.

Although the alternative answers may seem sensible to some, the classic definition is given in (a). This is the one of the requirements for the prediction that assets that appear to offer a higher expected return will be in demand, so their price will rise, so their expected return will fall because the expected return is a function of return and price.

Question 3
The diversification effect on the portfolio is greatest when the correlation between assets is lowest.
If a portfolio is assumed to comprise two components, which of the following correctly describes ‘correlation’?
(a) it is the covariance of the components divided by the product of the component variances
(b) it is the product of the component variances divided by the covariance of the components
(c) it is the covariance of the components divided by the product of the component standard deviations
(d) it is the product of the component standard deviations divided by the covariance of the components

Answer
The right answer is (c) it is the covariance of the components divided by the product of the component standard deviations.

Correlation is effectively a standardised version of covariance. (Covariance is itself a measure of how much two variables change together.) Because covariance is divided by the product of the component standard deviations, the correlation measure can only vary between +1 (perfect correlation) through zero (no correlation) to -1 (perfectly negative correlation)

Question 4
As International Treasurer one of your responsibilities concerns advising subsidiaries on their risk management activities. At present two subsidiaries are causing you some concern; Sub A has a mean exposure of £50m equivalent with a standard deviation of the weekly percentage change that has been estimated at 1.2%. Sub B has an exposure of £25m with a weekly standard deviation that you have estimated at 1.9%.

In trying to decide where you should concentrate your efforts, you have assumed that gains and losses on the exposures are normally distributed and estimated the following potential losses relating to the exposures. Which one is right?

(a) Potential losses will be less than £720,000 for Sub A and £902,500 for Sub B with 95% confidence.
(b) Potential losses will be less than £990,000 for Sub A and £783,500 for Sub B with 95% confidence
(c) Potential losses will be less than £1,398,000 for Sub A and £1,350,000 for Sub B with 99% confidence.
(d) Potential losses will be less than £1,980,000 for Sub A and £1,567,500 for Sub B with 99% confidence.

Answer
The right answer is (b) Potential losses will be less than £990,000 for Sub A and £783,500 for Sub B with 95% confidence.

Assuming that gains and losses on the exposures are normally distributed and that your estimates of standard deviation are reasonable accurate, then Value at Risk is given by:

\[
 \text{Standard deviation} \times \text{exposure} \times 1.65 \quad \text{(for 95\% confidence)}
\]

And:

\[
 \text{Standard deviation} \times \text{exposure} \times 2.33 \quad \text{(for 99\% confidence)}
\]

Answer (b) uses the 1.65 factor to correctly give the VaR for 95% confidence. Answer (a) incorrectly uses a different formula, involving the squaring of the standard deviation, and answer (d) assumes that the 99% confidence figure is just double the 95% confidence figure.
All of the answers indicate that the higher risk exposure is the in Subsidiary A.

**Question 5**
Value at Risk has been used extensively as an indicator of portfolio risk in recent years. However it relies on two key factors: the normal distribution of portfolio returns and a reasonable reliable estimate of standard deviation. Questions have been raised regarding the applicability of the normal distribution with the so-called ‘fat tails’ causing concern because they increase the probability of events occurring at the extremes of the distribution. Questions have also been raised that relate to the appropriate methodology for measuring standard deviation, or volatility.

Which of the following best describes the measurement of volatility?

(a) an average using market return data over the last 5 years
(b) an average using market return data over the last 5 months
(c) using market values of traded options to deduce implied volatility
(d) we don’t know the right number – it is the future that is important not the past

**Answer**
The right answer is (d) we don’t know the right number – it is the future that is important not the past.

The concern is that we need to estimate the extremes of what might happen in the future, and we can only use the evidence of the past. This may not be a good guide for the future!

**Question 6**
Pension scheme deficits have made headlines again recently. After a period of reducing deficits to a point where they were un-newsworthy, that situation has changed dramatically.

Imagine a pension scheme with liabilities and assets both equal at £1 billion. The asset portfolio comprises 70% equities and 30% bonds with a duration of 12 years. The liabilities represent the payments that must be made to pensioners in the future, with an overall duration of 30 years.

At the last valuation, the bonds in your asset portfolio were initially yielding the average AA bond rate of 5.10%, the same rate being used to value liabilities. Since that valuation the AA bond rate has fallen 30 basis points to 4.80%. In addition the equity market has fallen by 2%.

Which of the following gives the expected deficit that results from these assumptions?

(a) £27.3 million
(b) £47.2 million
(c) £89.4 million
(d) £134.6 million

**Answer**
The right answer is (c) £89.4 million

There are three stages to calculating the new deficit; calculating the value of each of the components.
First the equities: the investment is 70% of £1 billion and this has fallen by 2% to £686 million.
Second the bonds in the asset portfolio; modified duration is 12 / (1 + 5.1%) i.e. duration / (1+yield). The change in value is given by:
\[
\text{initial value} \times \text{modified duration} \times \text{change in rate} \\
= £300 \times (12 / 1.051) \times .003 = £10.276 \text{ million, so new bond value} = £310.276 \text{ million}
\]

Total asset value is then £686 + £310.276 = £996.276 million.

Finally the liabilities have increased in value as the AA bond rate has fallen. The calculation is as follows.

\[
\text{change in value} = \text{initial value} \times \text{modified duration} \times \text{change in rate} \\
= £1000 \text{ million} \times (30 / (1+5.1\%)) \times .003 \\
= £85.633 \text{ million} \\
\]

New liability value = £1,085.633 million

Therefore the new deficit = £1,085.633 million - £996.276 million
= £89.357 million