2.1.2 Investor Expectations and the Cost of Capital Self-Test Questions

Unit: Unit 1 – Treasury Strategy

Module: Module 2 – Risk and return

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Summary: Self test questions on investor expectations, CAPM and the cost of capital.

Keywords: security market line, covariance, correlation

Question 1

The capital asset pricing model is sometimes stated as follows:

Expected return = $\alpha + \beta r_m$

Where r_m is the expected return on the market

a) If α is zero, what must β be?
b) What must α be if β equals 2?
c) What must α be if β equals 0?
d) Give a general expression for α.

Question 2

True or false?

a) The capital asset pricing model implies that if you could find an investment with a negative beta, its expected return would be less than the risk-free rate.
b) The expected return on an investment with a beta of 2 is twice as high as the expected return on the market.
c) If the share lies below the security market line, it is undervalued.

Question 3

The return on a prospective investment has a standard deviation of 18%. The rate for Gilts is 7%, the market risk premium is 8%, and the standard deviation of market returns is 21%. What range does the beta of the investment lie within, and what range does this give for its required return?

Hint: Think about the definitions of covariance and correlation and how these are related.

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Answers

Answer 1
It is probably simplest to start with part d).

The usual form of the capital asset pricing model is

Expected return = $r_f + \beta(r_m - r_f)$

If we expand the bracket we can write this as

Expected return = $r_f + \beta r_m - \beta r_f = r_f(1-\beta) + \beta r_m$

Comparing this to the alternative form, we see that $\alpha = r_f(1-\beta)$

a) $\beta=1$, assuming $r_f \neq 0$
b) $-r_f$
c) r_f
d) $\alpha = r_f(1-\beta)$ as above

Answer 2
a) True
b) False. It offers twice the market risk premium.
c) False. The security market line shows all the possible combinations of the risk-free asset and the market portfolio which an investor can hold. All individual shares will be below this line.

Answer 3
Firstly, we remember that the correlation between two variables is defined as the covariance of the two variables divided by the product of their standard deviations. The correlation value can vary from -1 to +1. In this question, one of the variables is the investment, the other is the market portfolio.

$$\text{So, correlation} = \frac{\text{Cov (investment, market)}}{S_i \times S_m}$$

Where S_i is the standard deviation of the investment and S_m is the standard deviation of the market.

This is bounded by the values ±1, so we can write,

$$\pm S_i = \frac{\text{Cov(investment, market)}}{S_m}$$

Also, the beta of an investment is defined as the covariance between its return and the return of the market portfolio divided by the variance of the market portfolio returns.

So, $\beta = \dfrac{\text{Cov (investment, market)}}{S_m \times S_m}$

Therefore, substituting in,

$\beta = \dfrac{\pm S_i}{S_m}$

In this question the bounds for beta are therefore $\pm {}^{18\%}/_{21\%} = \pm {}^{6}/_{7}$

Using CAPM, the range for the required return is

from $r_e = 7\% - \frac{6}{7} \times 8\% = 0.14\%$

to $r_e = 7\% + \frac{6}{7} \times 8\% = 13.86\%$