4.3.1a Appendix A: Statistics

Unit: Unit 1 – Treasury Strategy

Module: Module 4 – Financial risk

Date: 1 March 2012

Summary: The behaviour of random variables can be described in terms of distributions, on which some powerful mathematical analysis can be deployed to answer many different types of question.

Keywords: random variable, expected value, mean, variance, standard deviation, distribution, normal distribution, central limit theorem, lognormal distribution, regression analysis, correlation

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Learning outcomes:

1. Random variables are the foundations of statistics
2. Random variables have an expected value and a variance and standard deviation
3. Date can form a frequency distribution
4. A normal distribution frequently describes many examples of variance in populations of data
5. The arithmetic of the normal distribution allows powerful predictions of ranges etc.
6. Other distributions are important in finance, especially the lognormal distribution
7. Correlation describes how variables might move together

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1 Introduction

Evaluation of risk often requires a statistical approach and this reading covers the basics required for risk management.

Probability refers to a degree of likelihood that something will happen, or that variables will take different values. In finance we are usually concerned with the probabilities associated with numerical ranges of important financial values like exchange rates, portfolio returns or the value of capital at the end of the next period. These are said to be *random variables*. This does not necessarily mean that they are completely unpredictable, but simply that, as of now, we don't know exactly what their future values will turn out to be.

A random variable is a variable whose value depends on the results of a random event.

A random event - for these purposes - is an event for which we know the following data:

- All possible 'outcomes' of the event that can happen.
- The probability of each 'outcome'.

Note that only one outcome of a random event can occur at a time. So if we want to know the probability of one of a number of (non-sequential) outcomes occurring in a random event, then this is just the sum of the probabilities of each outcome. (For example, the probability of rolling a one on a die is 1/6. The probability of rolling a two is also 1/6. The probability of rolling either a one *or* a two is 1/6 + 1/6 = 1/3.) This is an important concept to bear in mind when dealing with continuous random variables – see below.

There are two additional concepts to understand regarding random events:

- the expected value of a random event, the mean; and
- the variance and standard deviation of a random event.

We will initially be looking at the arithmetic mean: the average value that would occur if the random variable was observed many times. The arithmetic mean of n observations: x_1, x_2 … x_n is:

$$E[X] = \frac{1}{n}\sum_{1=1}^{n} x_i$$

The population variance is the mean of the **squares** of the differences between the arithmetic mean E[X] and each observation, x_i. The variance of n observations: x_1, x_2 … x_n is:

$$\text{Var}[X] = \frac{1}{n} x \sum_{1=1}^{n} (x_i - E[X])^2$$

Where the variance is calculated from a sample rather than from the complete population, the formula is modified. The denominator becomes n – 1 rather than n, so that the variance calculated from the sample then provides an unbiased estimate for the whole population variance:

$$\text{Var}[X] = \frac{1}{n-1}\sum_{1=1}^{n}(x_i - E[X])^2$$

The standard deviation is the square root of the variance.

Example 1: Calculating the mean, variance and standard deviation
Calculate the arithmetic mean, variance and standard deviation for the following population of random variables:

4, 2, 6, 1, 3, 8, 7, 2, 4, 10

Assume the population above is the total population.

Solution
The mean is just the arithmetic average of the numbers. To calculate the variance, we first take the deviations of each number from the mean, square them, add them, and finally divide by the number of observations, in this case 10. The standard deviation (often abbreviated to STD or σ) is the square root of the variance.

Exhibit 1: Calculating mean, variance and standard deviation

	x	x-E(x)	$[x-E(x)]^2$
	4	(0.70)	0.49
	2	(2.70)	7.29
	6	1.30	1.69
	1	(3.70)	13.69
	3	(1.70)	2.89
	8	3.30	10.89
	7	2.30	5.29
	2	(2.70)	7.29
	4	(0.70)	0.49
	10	5.30	58.09
Sum	47	0.00	78.10
Mean E(x)	**4.7**	**Variance**	**7.81**
		STD (σ)	**2.79**

The mean and the variance are obtained by dividing the sum for x and [x –E(x)]2 by the total population, 10.

The standard deviation (STD, σ) is the square root of the variance.

Note that the sum [x – E(x)] is zero as the sum of the deviations from the mean should equate to zero.

It is important to differentiate between random variables that are independent and those which are not independent.

Independent random variables are ones which do not affect each other. Knowing the value of one of the random variables does not provide any additional information about the other variable. (For example, if I were nimble-fingered I might simultaneously roll a die and toss a coin. Knowing the result of rolling the die - knowing for example that I had rolled a six - would not provide any additional information about the likelihood of getting heads or tails with the coin.)

An important property of normal random variables is that the sum of independent normal random variables is also a normal random variable. That is, if X_1 and X_2 are independent normal random variables with means μ_1 and μ_2 and with standard deviations σ_1 and σ_2, then X_1 plus X_2 is also normal with mean:

$$E[X_1 + X_2] = E[X_1] + E[X_2] = \mu_1 + \mu_2$$

And variance:

$$Var(X_1 + X_2) = Var(X_1) + Var(X_2) = \sigma_1^2 + \sigma_2^2$$

We will return to the properties of independent normal random variables later, after introducing the concept of the normal probability distribution function.

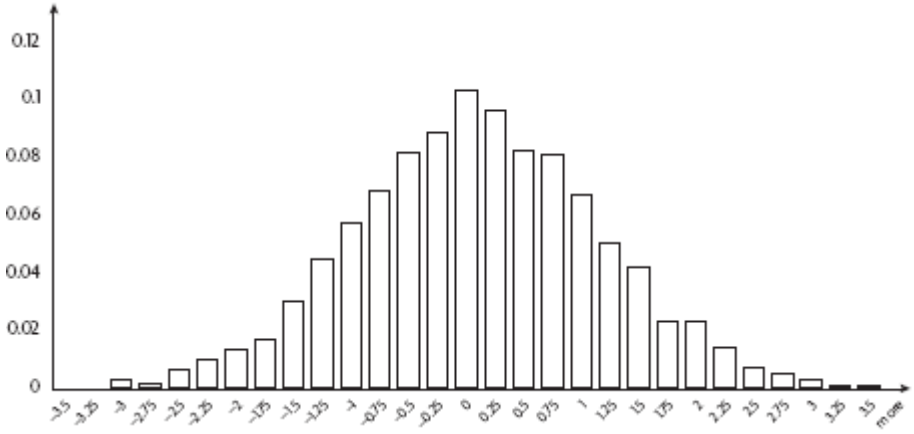
2 Probability and distributions

It is often useful to organise or arrange a body of data into a frequency distribution. This breaks up the data into groups or classes and shows the number of observations in each class. A histogram is a bar graph of a frequency distribution, where classes are conventionally measured along the horizontal axis and the related frequencies up the vertical axis. Given that in finance we deal a lot with data: prices, returns and values, this systematic way to describe the statistical properties of data series is very useful.

A *relative* frequency distribution is obtained by dividing the number of observations in each class by the total number of observations. The sum of the relative frequencies is equal to 1.

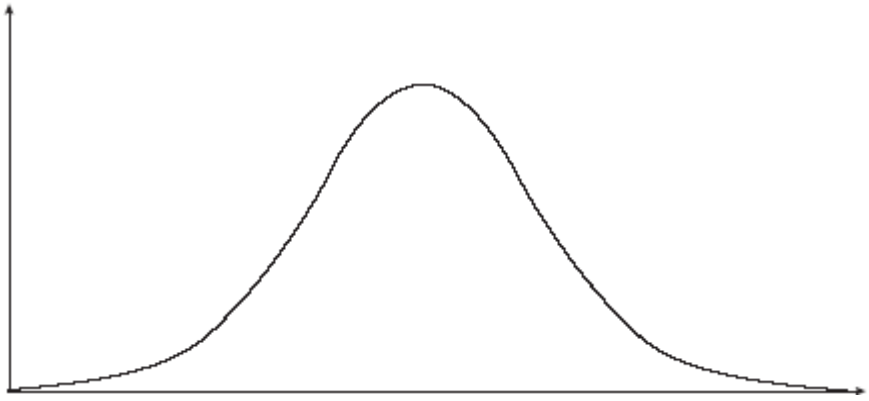
The histogram below is a probability relative frequency histogram of 5,000 numbers, ranging from roughly –3 to +3 with a tendency for the numbers to be near zero. The numbers are placed in boxes or bins, of width 0.25. Thus the height of the histogram in each bin is the number of observations in that bin as a proportion of the total 5,000.

Exhibit 2: Relative frequency distribution



What would happen if we increased the number of observations from 5,000 to infinity? In the present example, you would find that the distribution curve becomes smooth, illustrated below.

Exhibit 3: Continuous relative frequency distribution



The smoothness of the distribution curve is because the random variable concerned can take any value, i.e. is a continuous random variable. Many kinds of random variables are essentially discrete in nature: the number of breakdowns in a car fleet per year, average heights of a population and so forth. For discrete variables, the distribution function will be stepped rather than smooth.

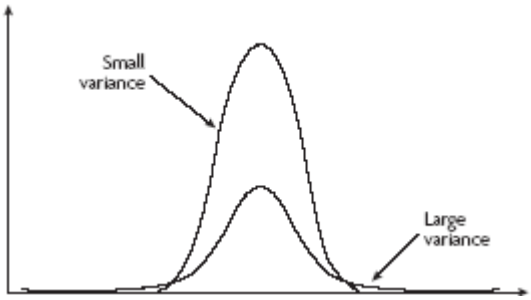
3 Normal distribution

The continuous distribution curve illustrated above is a special kind of continuous probability distribution function called a normal distribution. Its density function is bell shaped and symmetrical. The normal distribution can be defined by only two parameters, namely the mean and variance, because of its uniformity.

The maximum value of the normal distribution curve occurs when the random value, x, is equal to the mean, μ, the curve being symmetrical about the mean. The spread of the probabilities is determined by the variance. A small variance produces a sharp peak at x = μ, consequently most of the area under the normal curve is close to μ. A large variance, on the

other hand, produces a smaller, more rounded bulge at x = μ. The area under this normal curve is less concentrated about μ.

Exhibit 4: Two normal relative frequency distributions



The probability of the normally distributed random variable being below the mean is equal to the probability of the random variable being above the mean (both of these probabilities being 50%). The curve extends without limit in both directions but, as mentioned above, most of the area (probability) is clustered around the mean:

- 68.26% of the area (probability) under the normal curve is included within one standard deviation of the mean, within μ ± 1σ;
- 95.44% within μ ± 2σ; and
- 99.74% within μ ± 3σ.

The continuous probability distribution is a *standard* normal distribution if the mean is 0 and the standard deviation is 1. Rather than explain this in longhand every time that we describe a normal distribution, we can abbreviate it to the format φ (μ,σ). The symbol for a normal distribution function is φ. We use the notation φ (0,1) later in this reading to signify a standardised normal distribution.

Any normal distribution (X scale in Exhibit 5) can be transformed into a standard normal distribution by letting μ = 0 and expressing deviations from μ in standard deviation units (the horizontal z scale in Exhibit 5).

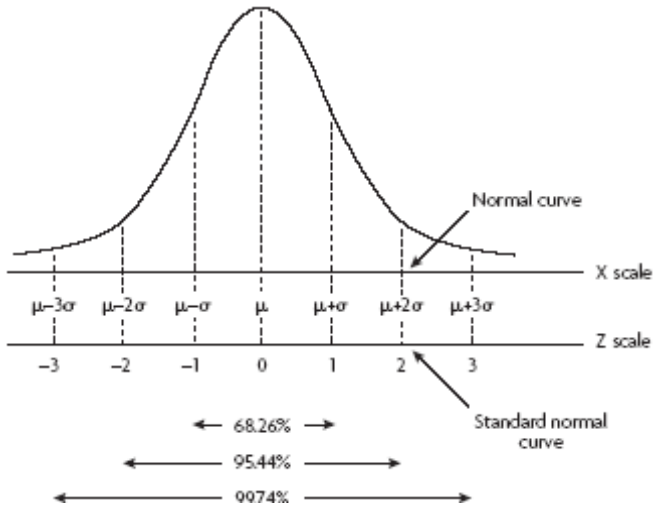
To find the probabilities (areas) for statistical problems involving the normal distribution, we first convert the X value into its corresponding z value, as follows:

$$z = \frac{X - \mu}{\sigma}$$

We can look up the z value in a standard normal distribution table. The z value is the number of standard deviations from the mean.

The following exhibit shows how a normal distribution function can be transformed into a standard normal distribution function.

Exhibit 5: Normal curve, standard normal curve



The table below shows a number of z values and the related probabilities of a result between the z value and the mean.

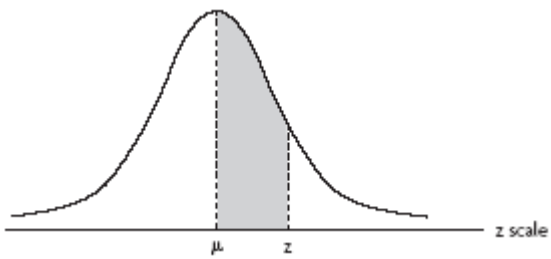
Exhibit 6: Probabilities (area) for the standard normal distribution

z	Probability
0.0	.0000
0.5	.1915
1.0	.3413
1.5	.4332
2.0	.4772
2.5	.4938
3.0	.4987

The following diagram shows the probability for one z value.

For example if z = 2.0, the shaded area is 0.4772 out of the total area of 1.0000.

Exhibit 7: Highlighting the area (probability) between μ and z for standard normal curve.



Note: Because the standard normal distribution function is symmetrical, the area between z = 0 and z = –2 will equate to the area between z = 0 and z = +2.

Example 2: Calculating probabilities using the standard normal distribution function
Suppose that X is a normally distributed random variable with μ = 16 and σ = 2, and we want to find the probability of X assuming a value between 14 and 18.

Solution

We first calculate the z values corresponding to the X values of 14 and 18 and then look up these values in a standard normal distribution table.

$$z_1 = \frac{X_1 - \mu}{\sigma} = \frac{14 - 16}{2} = -1 \text{ and } z_2 = \frac{X_2 - \mu}{\sigma} = \frac{18 - 16}{2} = +1$$

Looking at table 1, for z = 1, we get 0.3413. z = ±1 equals 2 x 0.3413 or 0.6826, this means that the probability of X assuming a value of between 14 and 18, or P(14 < X < 18) is 68.26%.

Example 3: Predicting rainfall using the standard normal distribution function
Suppose the annual rainfall in a city has a normal distribution with mean 40 inches, standard deviation 5 inches. What is the probability that the city will get between 40 and 45 inches of rain next year?

Solution
We want P(40 < z < 45) where z is the annual rainfall:

$$z_1 = \frac{X_1 - \mu}{\sigma} = \frac{40 - 40}{5} = 0 \text{ and } z_2 = \frac{X_2 - \mu}{\sigma} = \frac{45 - 40}{5} = +1$$

Thus we want the area (probability) between z1 = 0 and z2 = 1. Looking up z = 1 in Table 1, we get 0.3413. As z = 0 equates to a probability of zero, the probability of the annual rainfall being between 40 and 45 inches next year is 34.13%.

The mean is also known as a measure of 'central tendency'. There are five such measures in total but the mean is the most popular followed by the median, which is the value such that there are an equal number of observations above the median as below. For a normal distribution – or any symmetric distribution – the mean and median are the same, but this is not so for asymmetric distributions.

In the previous section, the concept of an independent random variable was introduced. If two random variables do not affect each other, they are independent.

They have the following important features:

Mean:

$E[X_1 + X_2] = E[X_1] + E[X_2] = \mu_1 + \mu_2$

And variance:

$Var(X_1 + X_2) = Var(X_1) + Var(X_2) = \sigma_1^2 + \sigma_2^2$

Suppose a random variable follows a standard normal distribution and that the change in its value in one year is φ (0,1)[1]. What is the probability distribution of the change in the value of the variable over two years, assuming that the two probability distributions are independent and otherwise identical?

The change in two years is the sum of two standard normal distributions, each of which has a mean of zero and a standard deviation of 1. When we add the two normal distributions, the result is a normal distribution in which the mean is the sum of the means and the variance is the sum of the variances.

The mean of the change over two years in the variable we are considering is therefore zero, and the variance of this change is 2.0. The probability distribution of the change in the variable over two years defined by the mean and standard deviation is therefore φ (0,√2).

A similar argument can be used to calculate the probability distribution of the change in the random variable over six months assuming that the change in the first six months is the same as the next six months and the probability distribution for the year is φ (0,1). The standard deviation of the change is √0.5 so that the probability distribution for the change in the value of the variable during six months is φ (0,√0.5).

Looking at a very short period of time δt, the probability distribution for the change in the value of the variable is φ (0,√δt) if the probability distribution for the year is φ (0,1). (For the non-mathematicians among you, δ is the symbol for a very small change in a variable; and Δ the symbol for a bigger, generally measurable, change.)

The reason for the square roots is that variances of successive time periods for independent and identical random variables are additive but the standard deviations are not.

Our calculations above made two important assumptions:

1. We assumed that the two distributions for successive periods of time were independent. This assumption is known as a "random walk".
2. We assumed that the distributions were otherwise identical. This includes a key assumption that there are no changes in the standard deviations between the successive periods.

[1] Mean of 0, standard deviation of 1.

4 Central limit theorem

The normal distribution is an important statistical distribution, not least because the sum of a large number of independent random variables, all having the same probability distribution, will itself be a normal random variable. As the sample size is increased, the sampling distribution of the **mean** approaches the normal distribution regardless of the shape of the parent population. The approximation is sufficiently good for values of **n** greater than or equal to about 30.

This is the *central limit theorem*, or 'law of large numbers'.

So as the sample size becomes very large, the distribution of the mean will be approximately the same as the normal distribution.

The central limit theorem is one of the ways of proving that share prices are lognormal. We discuss lognormal distributions in the next section.

5 Non-normal distributions

In real life, populations are distributions are seldom distributed normally, and other models have to be used to interpret and predict their behaviour. The easiest of these to get to grips with are lognormal, leptokurtotic and platykurtotic distributions.

5.1 Lognormal distributions

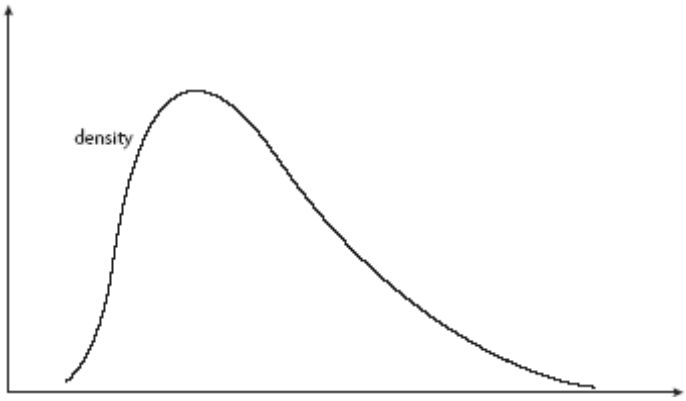
The probability density functions may not be symmetric; in other words, a random variable may not be normally distributed. For instance, the probability density function may have a longer tail on either side.

The exhibit below is a *lognormal distribution*. A random variable, X, has a lognormal distribution if the natural logarithm of the random variable, Y = ln(X), is normally distributed.

Note the longer tail on the right hand side, with a disproportionate probability of getting higher values. This is a *positively skewed density*.

Lognormal distributions are often used in financial modelling. This is because the lognormal distribution is an attractive one to assume in relation to share prices because it has a range of 0 to + infinity. This is exactly the theoretical range of asset prices because they cannot be negative but could attain very high positive values.

Exhibit 8: Distribution of a random variable which is lognormally distributed



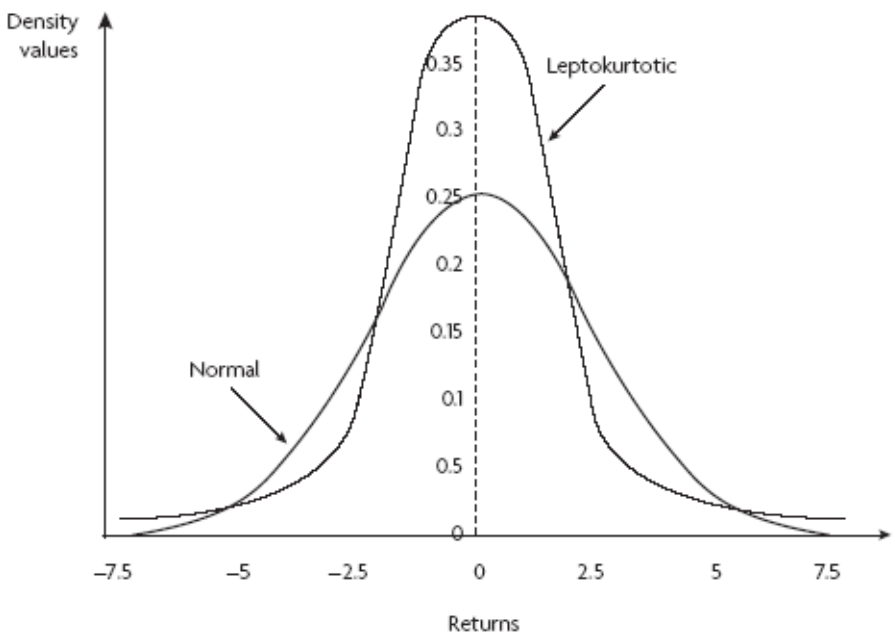
If the ratio of the price of the random variable at time t in the future to the present price is independent of the past history of prices, and has a lognormal probability distribution with parameters μ_t and σ_t^2 the series of prices for the random variable are said to follow a generalised Wiener process. This is one of the key assumptions in the Black Scholes option pricing model; the practical point for the treasurer is that implied, i.e. forward-looking volatilities are often quoted in the market on a lognormal basis, and this affects the calculation of VaR amounts.

5.2 Symmetric, but fatter or thinner than the normal

Even if the density is symmetric, it may not conform to the normal shape. Empirical distributions of returns often have fatter tails than predicted by the normal distribution, meaning a higher probability of getting very large or very small returns. They also tend to be more peaked around the centre. So you could describe them as fat tailed and peaky. The technical term is 'leptokurtotic'.

The exhibit below plots one particular leptokurtotic distribution against a normal distribution with the same variance.

Exhibit 9: Leptokurtotic density versus the normal distribution



Another type of kurtosis (i.e. the degree of difference from a normal distribution) is 'platykurtotic', which refers to a wide curve at the mean and very short tails.

6 Simple regression analysis

Suppose we have two random variables, x and y. They have their own arithmetic means, respectively μ_x and μ_y, and standard deviations, respectively σ_x and σ_y.

They will also have a co-variance, defined by:

$\sigma_{xy} = E[(x - \mu_x)(y - \mu_y)]$

If positive deviations $(x - \mu_x)$ for random variable x tend to be associated with positive deviations $(y - \mu_y)$ for random variable y, or negative with negative, then σ_{xy} will be positive, and we say that x and y are *positively correlated*. If, on the other hand, the positive deviations for x are associated with negative deviations for y, then x and y will have a negative co-variance and we say that x and y are *negatively correlated*.

If you want a measure of the closeness of fit of the proposed regression line, one statistical measure to use is the *co-efficient of correlation*, which is defined by:

correlation co-efficient $\rho = \dfrac{\sigma_{xy}}{\sigma_x \sigma_y}$

The most important property of the correlation co-efficient is that its value is always between –1 and 1. If x and y are statistically independent, their correlation co-efficient is zero. If $\rho > 0$ this means that the two variables are positively correlated so the regression line slopes upwards. If $\rho < 0$, the two variables are negatively correlated and the line slopes downwards.

The square of the correlation co-efficient (ρ^2 or r^2) measures the proportion of the variance of y which is explained by x. The greater r^2 is, the greater the strength of the linear relationship between x and y. If $r^2 = 1$, then we have a perfect fit.

Example 4: Calculating correlation co-efficient

Two securities, asset one and asset two, have standard deviations of 0.05 and 0.15 respectively, and a co-variance of –0.002. Find their correlation co-efficient.

Solution

The correlation co-efficient is: $\rho = \dfrac{\sigma_{12}}{\sigma_1 \sigma_2} = \dfrac{-0.002}{0.05 \times 0.15} = -0.27$

Note the negative signs. The negative correlation means a negative correlation co-efficient and this implies that the regression line slopes downwards.

7 Normal distribution table

Table A

Cumulative Distribution Function for the Standard Normal Random Variable [N(x)] where x ≥ 0.

The table shows values of N(x) for x ≥ 0. The table can be used with interpolation. For example: $N(0.4245) = N0.42) + 0.45 \times [N(0.43) - N(0.42)] = 0.663 + 0.45 \times (0.666 - 0.663) = 0.664$

Note that this table provides cumulative values of probability starting from 0.5, ie it gives the cumulative value for the entire distribution from –∞ to (mean plus Z standard deviations). The alternative version of the table (Table B below) provides the cumulative value for the "upper half" of the distribution only, ie it starts from 0.0 and gives the cumulative value for the half distribution from mean to (mean plus Z standard deviations).

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	0.500	0.504	0.508	0.512	0.516	0.520	0.524	0.528	0.532	0.536
0.1	0.540	0.544	0.548	0.552	0.556	0.560	0.564	0.567	0.571	0.575
0.2	0.579	0.583	0.587	0.591	0.595	0.599	0.603	0.606	0.610	0.614
0.3	0.618	0.622	0.626	0.629	0.633	0.637	0.641	0.644	0.648	0.652
0.4	0.655	0.659	0.663	0.666	0.670	0.674	0.677	0.681	0.684	0.688
0.5	0.691	0.695	0.698	0.702	0.705	0.709	0.712	0.716	0.719	0.722
0.6	0.726	0.729	0.732	0.736	0.739	0.742	0.745	0.749	0.752	0.755
0.7	0.758	0.761	0.764	0.767	0.770	0.773	0.776	0.779	0.782	0.785
0.8	0.788	0.791	0.794	0.797	0.800	0.802	0.805	0.808	0.811	0.813
0.9	0.816	0.819	0.821	0.824	0.826	0.829	0.831	0.834	0.836	0.839
1.0	0.841	0.844	0.846	0.848	0.851	0.853	0.855	0.858	0.860	0.862
1.1	0.864	0.867	0.869	0.871	0.873	0.875	0.877	0.879	0.881	0.883
1.2	0.885	0.887	0.889	0.891	0.893	0.894	0.896	0.898	0.900	0.901
1.3	0.903	0.905	0.907	0.908	0.910	0.911	0.913	0.915	0.916	0.918
1.4	0.919	0.921	0.922	0.924	0.925	0.926	0.928	0.929	0.931	0.932
1.5	0.933	0.934	0.936	0.937	0.938	0.939	0.941	0.942	0.943	0.944
1.6	0.945	0.946	0.947	0.948	0.949	0.951	0.952	0.953	0.954	0.954
1.7	0.955	0.956	0.957	0.958	0.959	0.960	0.961	0.962	0.962	0.963
1.8	0.964	0.965	0.966	0.966	0.967	0.968	0.969	0.969	0.970	0.971
1.9	0.971	0.972	0.973	0.973	0.974	0.974	0.975	0.976	0.976	0.977
2.0	0.977	0.978	0.978	0.979	0.979	0.980	0.980	0.981	0.981	0.982
2.1	0.982	0.983	0.983	0.983	0.984	0.984	0.985	0.985	0.985	0.986
2.2	0.986	0.986	0.987	0.987	0.987	0.988	0.988	0.988	0.989	0.989
2.3	0.989	0.990	0.990	0.990	0.990	0.991	0.991	0.991	0.991	0.992
2.4	0.992	0.992	0.992	0.992	0.993	0.993	0.993	0.993	0.993	0.994
2.5	0.994	0.994	0.994	0.994	0.994	0.995	0.995	0.995	0.995	0.995
2.6	0.995	0.995	0.996	0.996	0.996	0.996	0.996	0.996	0.996	0.996
2.7	0.997	0.997	0.997	0.997	0.997	0.997	0.997	0.997	0.997	0.997
2.8	0.997	0.998	0.998	0.998	0.998	0.998	0.998	0.998	0.998	0.998
2.9	0.998	0.998	0.998	0.998	0.998	0.998	0.998	0.999	0.999	0.999
3.0	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999
3.1	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999
3.2	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999
3.3	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
3.4	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

Table B

Cumulative Distribution Function Z for the Standard Normal Random Variable [N(x)] where x ≥ μ.

The table shows values of N(x) for x ≥ μ. The table can be used with interpolation.

The values in this table should be equal to the values in Table A less 0.5. There are small differences due to rounding.

Z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549
0.7	0.2580	0.2611	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2967	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
2.9	0.4981	0.4982	0.4982	0.4982	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990

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