2.1.1 Investor Expectations and the Cost of Capital[1]

Unit: Unit 1 – Treasury Strategy

Module: Module 2 – Risk and return

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Summary: An introduction to the theory of expected returns for the investor and the cost of capital for the firm.

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[1] This reading includes an activity requiring you to post in the online forum. This activity is indicated by the icon. You are expected to post forum activities to the appropriate forum within the two-week timeframe of the module. Please see the study timetable for reference.

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Learning outcomes:

1. To understand why only market risk is rewarded.
2. To have a detailed understanding of the theory underpinning the Capital Asset Pricing Model.
3. To have an appreciation of some of the other asset pricing models.
4. To have a detailed understanding of how the cost of equity is determined.
5. To understand the limitations of beta.
6. To Understand how the cost of debt is determined, in particular the use of the marginal tax rate
7. To understand how WACC is calculated.

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1 Introduction

A firm that is established in a particular sector, which let us assume most firms with conventional financing are, has made the broad decision about where to invest funds. Thus an airline invests in aircraft, routes, ground facilities, staff etc. But after this initial broad choice, there is a continuing round of decision making about where to invest the capital it has earned from its operations (retained profits) and raised from its shareholders and lenders. Firms must invest to keep their assets and ideas up to date, so an airline will invest in more modern aircraft for its core routes on a regular basis. But they will also usually try and grow or become more profitable, by investing in new routes (or scrapping unprofitable ones) or improving the infrastructure. This investment requires a financial test of some sort.

Leaving aside discounted cash flow techniques for the moment, we can assume that firms only invest in projects which they expect will make a sufficient profit after all costs, including interest on any borrowings and taxes on profits are taken into account. What is left belongs to the *owners* of the i.e. shareholders. This is the case whether the profits are distributed to the shareholders as dividends or retained within the firm to be invested in other projects.

However, what return should the company be giving shareholders?

In most arms-length transactions, there is a willing buyer and a willing seller. The forces of supply and demand balance these out and an exchange happens at an agreed price.

In company financing terms this is true of bank borrowings where the agreed price is the interest rate.

This is not quite as simple on the equity side, although similar mechanics are in operation. This time the return is not agreed up front, but there is an expectation of a return. Investors only invest if they expect the firm to deliver adequate returns, i.e. the firm meets their expectations. Thus the cost of equity to the firm is the same as the return an investor expects to receive.

So, going back to our initial question, a firm should invest in those projects or buy those companies which generate a return that meets or exceeds the returns required by its lenders and expected by its investors.

We therefore need to establish what return equity investors require.

We use the Capital Asset Pricing Model to price the cost of equity and this reading takes you through the theoretical underpinning.

Since most firms are financed by a mix of equity and debt, the cost of capital for such firms is a blend of these two components and is commonly called the Weighted Average Cost of Capital (WACC). This reading builds to a detailed examination of this concept.
Further reading, not compulsory

Read Chapter 3 (The Basics of Risk) and Chapter 4 (The Cost of Equity) from Applied Corporate Finance, by Aswath Damodaran, 4th edition.

We recommend that you work through this reading before looking at Damodaran, although you may prefer to work through both at the same time. Damodaran's book should help to deepen your understanding of the concepts introduced in this reading, particularly by relating them to real-life situations.

2 Equity risk and returns

Equity risk exists because of the possibility that the actual returns shareholders receive will turn out to be less than expected.

The dominant theory relating risk and returns is the Capital Asset Pricing Model, but before we can talk about this we need to understand some key concepts – in particular how we measure risk, diversification and the market portfolio.

3 How do we measure equity risk?

In financial markets, risk to the investor is the likelihood of actual outcome differing from expected outcome.

Later in this reading we will talk about the risk free asset. This is usually characterised as a government bond since an investor who buys such a bond at any time in its life and holds it to maturity is certain of its return, principally because a government can always 'print' the money to repay the bond.

3.1 Forum discussion

Is a government bond really a risk free asset? You might like to think of dimensions of this risk including Eurozone members, quasi governmental borrowers such as cities, states or counties in the US, local authorities in the UK and similar constructions worldwide.

Post your thoughts to the module forum and respond to others[2]: create a new post in the *Forum exercise from 2.1.1 Investor expectations (risk free asset)* category and title it U1/M2/2.1.1 Risk free asset /[Your name].

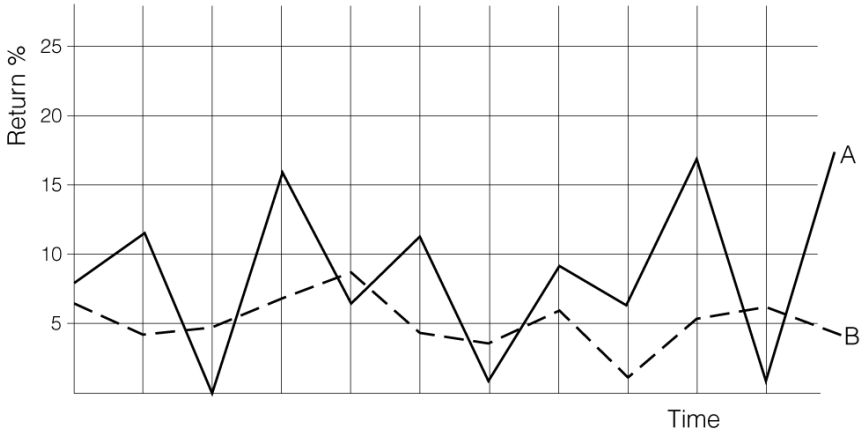
However, the actual return on most risky assets will differ from the expected return. The higher this potential difference, the riskier the asset.

We measure this share-price volatility (or to use the London Business School term, **variability**) using standard deviation, the spread of individual returns around the mean. In the diagram below, share A is clearly more volatile and therefore more risky than share B.

[2] You are expected to post forum activities to the appropriate forum within the two-week timeframe of the module. Please see the study timetable for reference.

3.2 Volatility of returns

Exhibit 1: Volatility of returns



The volatility of the price of a share can easily be quantified by calculating the standard deviation of the share returns for a given period. The period can be as short as a few minutes to as long as years.

Example 1: Calculation of Standard Deviation

	Returns %	Deviation from the mean	Deviation squared
	16	4	16
	5	(7)	49
	32	20	400
	(8)	(20)	400
	11	(1)	1
	20	8	64
	5	(7)	49
	9	(3)	9
	16	4	16
	14	2	4
Totals	120	0	1008

$$\text{Mean} = \frac{\text{Sum of Returns}}{\text{Number of measurements *}} = \frac{120\%}{10} = 12\%$$

* Strictly, as we are using a sample, we should use n - 1

$$\text{Variance} = \frac{\text{Sum of the squares of the deviations from the mean}}{\text{Number of measurements}} = \frac{1008\%}{10} = 100.8\%$$

$$\text{Standard Deviation} = \sqrt{\text{Variance}} = 10.04\%$$

The risk of the share is the extent to which the actual return might fall above or below the mean expected return. In the above example, we might reasonably expect returns to average 12% with a standard deviation of 10.04%. This means that in any one of our chosen periods we could expect the return to be within one standard deviation of the mean on 68 out of 100 occasions. This is quite a range, the return could fall to 2% or rise to 22%. For the other 32 occasions, we could expect the return to be outside these parameters.

If the returns are "normally distributed", i.e. following the normal distribution (bell-shaped) curve shown below, the standard deviation can be used to quantify given areas under the distribution curve, which can be translated into the probability of a particular return occurring. Approximately 68% of the area of a normal distribution lies within 1 standard deviation either side of the mean, so the probability of a return higher or lower than 1 standard deviation is approximately 16% in each case (i.e. half of the difference between 100% and 68%). The probability of a return more than 3 standard deviations either side of the mean is virtually zero (0.27%).

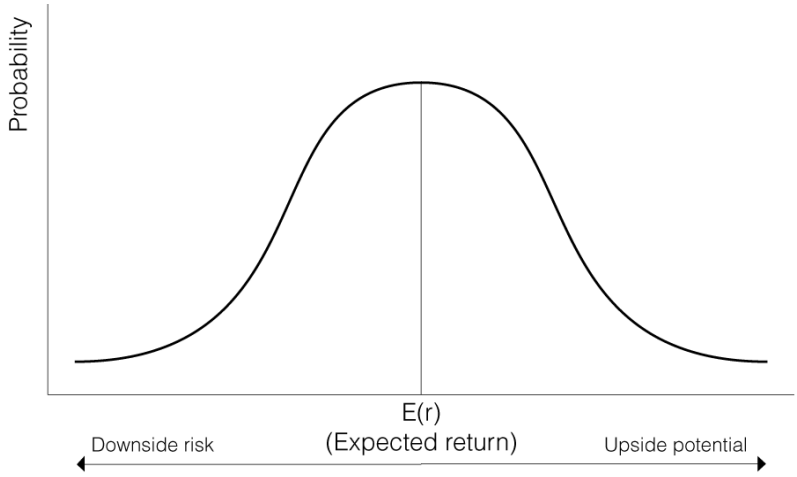
- Range +/- 1 sd Probability within 68.27%. Downside risk 15.87%
- Range +/- 2 sd Probability within 95.45%. Downside risk 2.28%
- Range +/- 3 sd Probability within 99.73%. Downside risk 0.14%

These can also be expressed in a slightly different way:

95% of all outcomes will be within +/- 1.96 standard deviations of the mean, and 99% will be within +/- 2.56 standard deviations.

3.3 Normal distribution

Exhibit 2: The Normal Distribution



Example 2: Calculations around normal distributions
Elasticity Inc has some funds to invest and has been given the following guidelines:

The minimum acceptable return is 8% but returns less than this are acceptable if the chance of those lower returns is 2.5% or less.

Elasticity is reviewing one investment with expected returns (actually their historical average) of 13% and with standard deviation, or volatility, of returns of 3%.

It turns out that this investment is unsuitable because there is a 2.5% chance that it could return 7.12% (13% less 1.96 x 3% = 7.12%) and therefore a probability in excess of 2.5% of returns of 8% or lower.

Elasticity needs to find an investment with the same volatility but with an expected mean of 13.88% or higher or an investment with an expected return of 13% and a standard deviation of 2.55% or lower.

The validity of the normal distribution
It is well known that asset returns do not behave according to the normal distribution and underestimate extreme losses. The stock market crashes of 1929 (Wall Street Crash), 1987 (Black Monday), 2000 (Dot Com Bubble), and most recently the 2008 Financial Crash are all extremely unlikely events under the normal distribution. These are examples of Taleb's "black swan" events. It is often suggested that a flatter distribution with fatter tails (kurtosis) is more appropriate. It is also known that for longer holding periods, e.g. 12 months or more, the distribution tends to be skewed to the right, yet the normal distribution is symmetrical.

In many instances, however, a normal distribution can be assumed, since it approximates many actual distributions for financial phenomena.

4 Diversification

One of the main tenets of corporate finance theory is that the variability of share price returns can be divided into **market risk** and **firm specific risk.** We will demonstrate that an investor should only be rewarded for his exposure to market risk.

4.1 Firm specific and market risks

When a firm makes an investment, it takes on risks as the return on the investment is not within its direct control. These risks can be analysed as:

- **Firm-specific risks** (also known as **unsystematic risk** or **diversifiable risk**):
  - project specific risk
  - competitive risk
  - industry specific risk
  - international risk
- **Market risk** (also known as **systematic risk** or **non-diversifiable risk**)

Firm-specific risks

Project specific risk

This is simply the risk that the project may have different cashflows than expected. In most firms undertaking several projects, you would expect some projects to under-perform and others to over-perform, so part of this risk is diversified away within the firm.

Competitive risk

This is the risk which arises from the actions of competing firms. Whilst good project analysis should take competitor action into account, the firm cannot diversify away this risk. However, investors can, by holding the shares of competitors.

Example 3: Competitive risk

Banana Inc and Robot Inc make software for mobile phones and each also has investments in different parts of the supply chain for the phone. However, much innovation is in the software and each takes turns to leapfrog the other in terms of product capability. Their turnover and profit alternately go up and down.

Canny Counters Fund Managers like the technology sector, and mobiles in particular, and so buy shares in Banana and Robot in equal values, thus removing most competitive risk from their portfolio.

If Parsnip plc are successful with their own version of the software, and take the lead, however, then Canny Counters will not have managed the removal of this risk.

Industry specific risk

These are factors which affect a specific industry and can be further analysed into:

- technological risk – new or evolving technologies
- legal risk – changing laws and regulations
- commodity risk – affecting the price of goods and services

Again, the firm cannot easily diversify away from these risks, but an investor can by holding shares in different industries.

International risk

Any firm with exposure to exchange rates (or indeed international competition etc.) will be subject to this risk. Many will also be exposed to other political risks as well. A firm can partially diversify away from exchange rate risk by spreading its exposure to different currencies or by asset matching, but it is easier for investors to do this by holding global portfolios.

Market risk

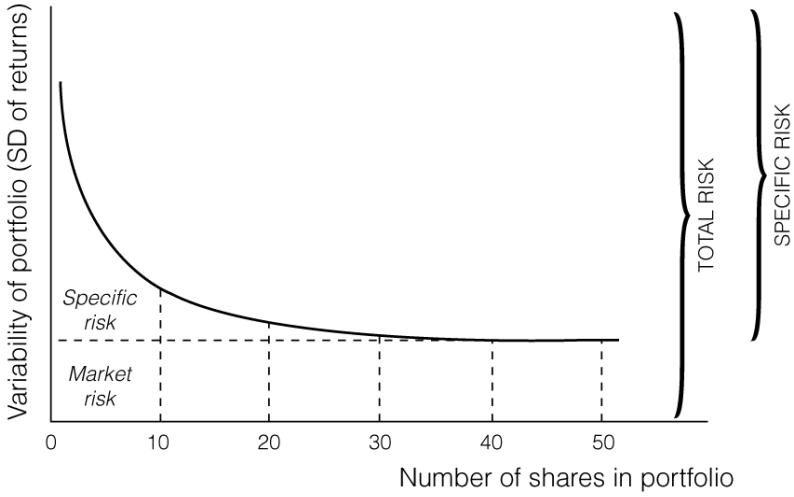
These are the macroeconomic risks which affect all firms, although not necessarily to the same extent, such as interest rates, inflation and GDP growth. Neither firms nor investors can diversify away from market risk as all risky investments are exposed to these risks to some extent.

4.2 Diversification

Firms can diversify risk themselves over and above the project-specific risk discussed above. However, investors can achieve the same diversification simply by holding shares in different sectors and, most importantly, at a lower cost.

Research has shown that shareholders are able progressively to diversify away firm specific risk by holding an increasing number of shares in their portfolio. This is illustrated in the diagram below where, at the limit, there is only market risk left. The ultimate limit is to hold all the shares in the market, weighted in proportion to their market capitalisation. This is called the market portfolio.

Exhibit 3: The benefits of diversification



4.3 The importance of diversification?

Consider two investors, one more diversified than the other. Both investors need rewarding for market risk, but the less diversified investor will need rewarding for a higher proportion of firm specific risk. Therefore, the more diversified investor will be willing to pay more for the investment (or the less diversified investor will diversify to achieve higher returns). Logically all investments will end up in the hands of diversified investors as they are able to pay the highest prices. Theoretically investments will end up in the hands of the fully diversified investor who has no exposure to firm specific risk. Such an investor is only rewarded for his exposure to market risk.

5 Modern portfolio theory (MPT)

The essentials of MPT were first set out by Markowitz in 1959. MPT provides the intellectual framework that underlies the link between risk and return.

The model makes a number of simplifying assumptions, but in particular:

1. Investors prefer a greater return to a lesser return.
2. Investors prefer less risk to more risk.
3. Investment risk is characterised by the normal distribution

Example 4: Modern Portfolio Theory

Consider 4 investment portfolios with the following levels of return and risk:

	Expected return (%)	Risk/ Std Deviation (%)
A	23	32
B	21	28
C	25	32
D	23	28

B's risk is the same as that of D at 28%, but D has a higher rate of return, 23% as opposed to 21%, and is therefore preferable.

D also has the same rate of return as A, but has a lower level of risk, and so is again preferable.

You should also be able to see that C is preferred over A, because it has a higher return at the same level of risk.

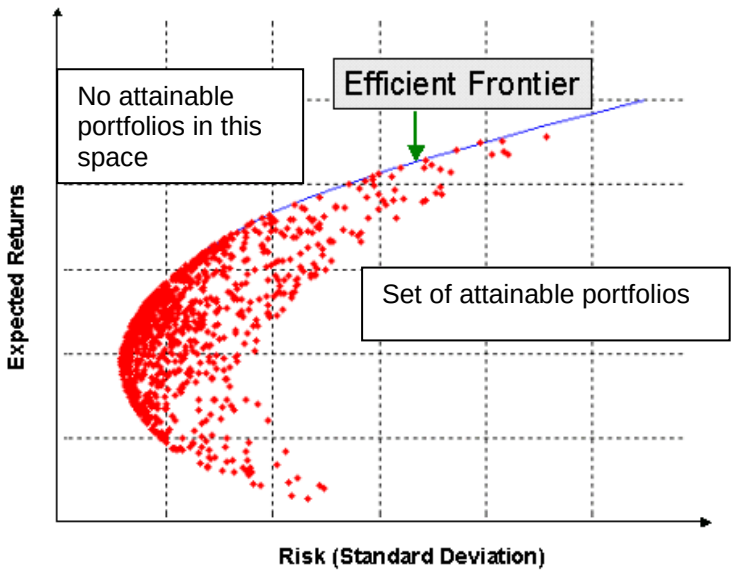
C and D are therefore dominant in that they offer higher returns for a given level of risk or lower risk for a given return.

5.1 The efficient frontier

Now, imagine that this process has been carried out for all the portfolios on the stock market. The diagram below shows what the result might look like if all these portfolios have been plotted as red dots. (Note that all individual share returns could also be plotted, effectively as single share portfolios.)

The portfolios which comprise those offering the highest return for each level of risk make up the **efficient frontier** (the bold blue curve). The efficient frontier is curved as investors will find there are diminishing returns for the additional risk taken on. Portfolios which lie on the efficient frontier are called **efficient portfolios**.

Exhibit 4: The efficient frontier

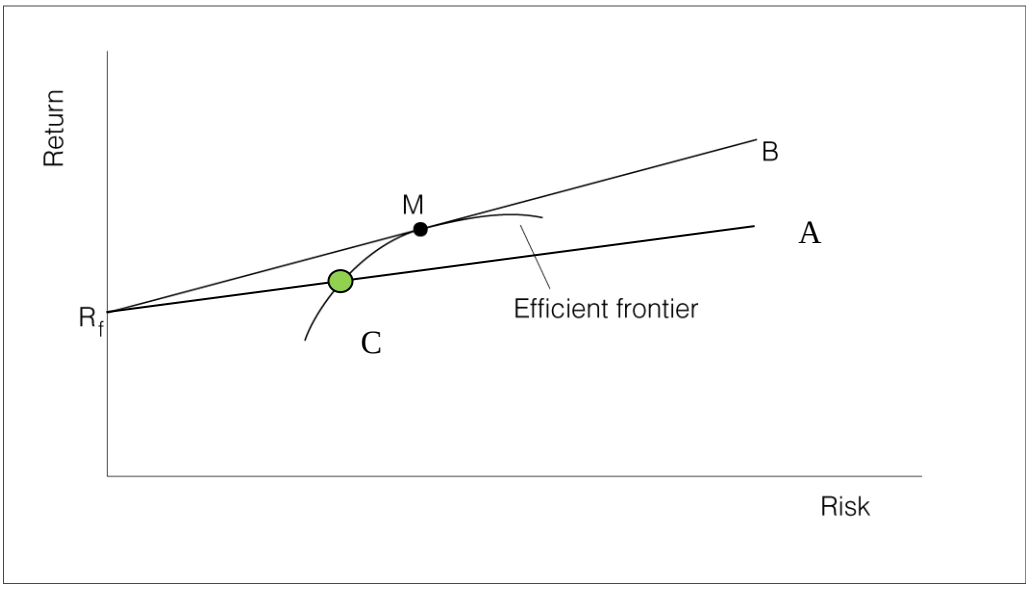


5.2 The addition of risk free assets

We now allow investors to borrow or lend at the risk free rate, the rate of interest on government securities.

If an investor invests some of his money in government securities and the balance in the efficient portfolio at point C (defined by the green dot in Exhibit 5 below,) he can vary his return from the risk free rate, R_f (if he fully invests in the risk free asset) to the return on the portfolio at point C (if he fully invests in the portfolio). The investor can achieve any return along the line R_f C. If the investor borrows at the risk free rate, he can increase his return further along the line CA.

Exhibit 5: The market portfolio



However, with the exception of a risk free portfolio, the investor can always increase his return without increasing his risk by investing in combinations of the risk free asset and the portfolio defined by the point M. This is where the line R_fB is tangential to the Efficient frontier.

With the introduction of the risk-free rate we get the result that there is only one optimal portfolio for all investors. This must be the **market portfolio** because no other portfolio can be constructed that has a better risk-reward relationship.

Various combinations of the market portfolio M and the risk-free asset (R_f) can be constructed to suit the investor's risk appetite. The line R_fB is known as the capital market line ("CML") or securities market line ("SML") and all portfolios that lie on this line are called *efficient*.

The implications of this model are that investors should all hold the market portfolio but with different amounts of risk-free borrowing or lending.

6 Asset pricing models

At MCT and in the finance world at large, the most commonly used asset pricing model is the capital asset pricing model ("CAPM"). However, students should be aware that there are alternatives and we will give a brief overview of these as well.

It is also worth remembering that these are all just models. They are by no means perfect and make various assumptions. However, neither does this mean that they are worthless.

6.1 The capital asset pricing model (CAPM)

CAPM builds on market portfolio and assumes that investors only invest in the market portfolio. This means that investors hold every traded risky asset in proportion to their market value. *Note that theoretically risky assets are not limited to shares, but also include bonds and real property amongst others.*

CAPM also assumes that investors allocate their wealth between risk free assets (where the return is the **risk free rate, denoted by R_f**) and the market portfolio (which requires a return in excess of the risk-free rate to compensate for the extra risk; the so-called **market risk premium, denoted by R_m-R_f** where R_m is the return required for holding the market portfolio). A risk averse investor will invest primarily in risk free assets (where the return is guaranteed, such as government bonds) whereas more adventurous investors will invest the bulk of their wealth in the market portfolio. The model also allows investors to borrow at the risk free rate (the return on the risk free asset) so that they can take on even more exposure to the market portfolio.

6.1.2 How to apply CAPM

We can view the risk of a single asset as the additional risk when added to the investor's portfolio of investments.

Since the investor is well diversified, the additional risk will only relate to market risk as firm specific risk has already been diversified away.

This risk will either be the same as the overall market, lower or higher. We quantify this by calculating **beta** (normally denoted by **β**), where:

$$\text{Beta}(\beta) = \frac{\text{Covariance of the single asset with the market portfolio}}{\text{Variance of the market portfolio}}$$

Example 5: Calculation of beta

To calculate beta for a particular share, we measure the return for that share and the return on the market on 10 occasions, leading to the following calculations:

	Returns		Deviation from the mean		Product of the deviations from the mean	Market deviation squared
	Company	Market	Company	Market	Company x Market	
	16	10	4	(1)	(4)	1
	5	10	(7)	(1)	7	1
	32	16	20	5	100	25
	(8)	(24)	(20)	(35)	700	1,225
	11	5	(1)	(6)	6	36
	20	11	8	0	0	0
	5	21	(7)	10	(70)	100
	9	27	(3)	16	(48)	256
	16	19	4	8	32	64
	14	15	2	4	8	16
TOTALS	**120**	**110**	**.0**	**0**	**731**	**1,724**

$$\text{Company Mean} = \frac{120\%}{10} = 12\% \qquad \text{Market Mean} = \frac{110\%}{10} = 11\%$$

$$\text{Covariance} = \frac{\text{Product of the deviations from the mean}}{\text{Number of measurements}} = \frac{731}{10} = 73.1\%$$

$$\text{Market Variance} = \frac{1{,}724}{10} = 172.4\%$$

$$\text{Beta} = \frac{\text{Covariance}}{\text{Market Variance}} = \frac{73.1\%}{172.4\%} = 0.424$$

Typically beta varies between 0 and 2, although it can be higher. The beta for the market is 1 as the market portfolio, by definition, will deliver the market return. The beta for the risk free asset is 0 as the return is not affected by movements in the market portfolio.

Now, the investor will require an additional return for this additional risk, and this can be expressed as follows:

Additional Return = Return on the asset - $R_f = \beta(R_m - R_f)$

Rearranging:

Return on the asset = $R_f + \beta(R_m - R_f)$

6.2 Arbitrage pricing theory ("APT")

Arbitrage Pricing Theory describes a mechanism used by investors to identify an asset which is incorrectly priced.

Remember that the only valid corporate finance reason that an asset is purchased is to earn a return for a certain amount of risk—no other aspect of the asset matters. So, if any two assets have the same return-risk profile, they should sell for the same price – the Law of One Price. If this is not true, then a profit could be made by selling short the asset or portfolio with the lower return, and buying the higher return portfolio. This is arbitrage.

If the expected risk premium for an asset is lower than the calculated risk premium using the APT formula, then investors would sell the asset forcing down the price and increasing the return. On the other hand, if the risk premium were higher than the calculated value, then investors would buy the asset, pushing up the price and reducing the return. Investors would do this until the expected risk premium and the calculated risk premium were in balance.

Whereas the CAPM begins with an analysis of how investors construct efficient portfolios, APT assumes that the price of an asset is entirely driven by two groups of factors:

- market or macro economic factors, and
- firm specific factors or noise

As with the CAPM, firm specific factors can be ignored for diversified investors.
APT states that the expected risk premium on an asset should depend on the expected risk premium associated with each factor (f_i - r_f) and the asset's sensitivity (b_i) to each of these factors.

So, expected risk premium = $r - r_f = b_1(f_1 - r_f) + b_2(f_2 - r_f) + \ldots$

Example 6: Arbitrage pricing theory
Consider 2 portfolios, X & Y, each with exposure only to one factor, F. Portfolio X is two times as sensitive to F compared to portfolio Y. The APT formula implies that the expected risk premium for portfolio X must be twice that of portfolio Y. So, if an investor puts half his money into risk free assets and the other half into portfolio X, this combined portfolio would have the same sensitivity to F as portfolio Y and would offer the same risk premium.

Suppose that the APT formula does not hold. That is, there exists another combination of portfolio X and the risk free asset which offers a higher return. In this case investors could make an arbitrage profit by selling portfolio Y and investing in the higher returning portfolio. Clearly this should not be possible if the assets are correctly priced.

6.2.1 Factors used in the arbitrage pricing theory

The theory does not tell the investor what the factors are for a particular asset, and for a very good reason. In practice, and in theory, one asset might be more sensitive to one factor than another. For example, oil major shares will probably be very sensitive to crude oil prices, whereas technology shares may not be affected at all.

APT leaves it up to the investor to identify each of the factors for a particular asset. So, the investor has to identify three items:

- the factors affecting the asset
- the expected returns for each of these factors
- the sensitivity of the asset to each of these factors

Identifying and quantifying each of these factors is no trivial matter, and is one of the reasons why the CAPM remains the dominant theory.

6.3 Multifactor models

Essentially multifactor models are a special case of APT except that the factors are specified.

Multi-factor models can be divided into three categories: macroeconomic, fundamental and statistical models. Macroeconomic models compare an asset's return to such factors as employment, inflation and interest. Fundamental models analyse the relationship between an asset's return and its underlying financials (such as earnings). Statistical models are used to compare the returns of different assets based on the statistical performance of each asset.

6.4 Proxy models

CAPM, APT and multifactor models all consider market risk in economic terms and develop theories accordingly. There is a final set of models which analyse past returns on shares and then try to explain differences in returns by identifying firm characteristics.

The best known proxy model is probably the **Fama and French 3 Factor Model**. Fama and French started with the observation that two classes of shares have tended to do better than the market as a whole:

- small caps, and
- value stocks, ie shares with a high book-value-to-price ratio

Fama & French adjusted the CAPM by adding in 2 further factors to adjust for their observations. ***This is described in a little more detail on pages 72 - 73 from Applied Corporate Finance, by Aswath Damodaran, 4th edition.*** Table 3.3 on page 75 is also useful.

7 The cost of equity capital for the firm

We know that the required return to shareholders is given by CAPM, where the required return = $R_f + \beta(R_m - R_f)$. This is what the shareholders expect to receive, so this – as we explained in the Introduction – must be the cost of equity to the firm.

Some of you may be wondering if this applies to all shareholders. Do different shareholders require different returns? This would have interesting consequences for the cost of equity to the firm.

Generally, the shareholders will have purchased their holding at different times and so their actual returns will all be different. The key is the expected return based on the current share price.

Assuming a new investor has no inside information, we would expect him to buy a company's shares only if he is satisfied with the expected return based on the likely purchase price. However (ignoring tax and transaction cost considerations) we can also assume that all existing owners of the company's shares are also satisfied with the expected return based on the current share price. Otherwise a rational investor would sell the shares.

Therefore, when we talk about the cost of equity for the firm, *we assume that all shareholders hold the stock because they are satisfied with the expected return based on the current price*.

The expected return based on the current share price is therefore the cost of equity (K_e) for the firm if it is to meet shareholder expectations. The firm has to deliver this by way of dividends or capital growth in the share price.

So, $K_e = R_f + \beta(R_m - R_f)$.

The cost is post-tax because dividends are paid from taxed profits and earnings (profits after tax) are the driver for share price increases.

What about R_f, R_m & β?

7.1 The risk free rate

This is usually taken to be the interest rate (yield to maturity) on government securities. This can be regarded as risk free for domestic investors who buy an issue and hold to maturity. Most practitioners tend to use the ten year bond, where it exists, probably because that might be the average life of an investment but also because the ten year rate is generally seen as a major liquid benchmark.

7.2 The market return

We are really more interested in the market risk premium, the extent to which the market return exceeds the risk free return.

This varies significantly from period to period and from market to market. The result is that it is possible to find advocates for any figure between 0% and 10%, but with a long-term average across major markets of 4.5% to 6.5%.

However as the cost of equity is what the investor requires in the future, it is hard to see how historical analysis can provide this answer. Historical analysis is helpful, but in the period following the Financial Crisis for example, it was not really appropriate.

7.3 Beta

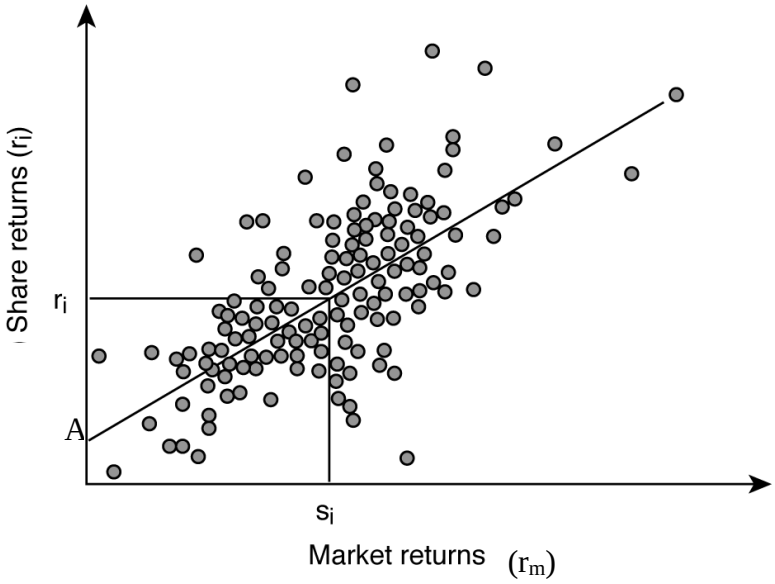
We saw from our work on CAPM that beta is defined as follows:

$$\text{Beta}(\beta) = \frac{\text{Covariance of the single asset with the market portfolio}}{\text{Variance of the market portfolio}}$$

Note that, as with the market premium, the investor is primarily interested in future volatility, yet most models rely on measuring past volatility.

The same statistical procedure is used to calculate the least squares regression line of best fit through a plot of the data points. This is demonstrated graphically on the scatter diagram below. Beta is the gradient (slope) of the regression line or line of best fit.

Exhibit 6: Beta as the gradient of the regression line



The regression line is a straight line with equation; $r_i = A + \beta r_m$

r_i is the rate of return on the share; r_m is the return on the market; β is the coefficient of variation between the share and the market portfolio; and A is often referred to as the abnormal return, although this is not correct – we shall discuss this further below.

7.3.1 Accuracy of beta

The beta itself does not, however, indicate any accuracy of fit. From the above scatter diagram, it is quite difficult to visualise where to draw the line of best fit and when this is the case, the line of best fit is not very predictive about the data points.

Note that there will always be a *mathematical* line of best fit, even if it is not obvious from observation. If the line is not obvious we say that the correlation is low.

Correlation is basically a standardised measure of the covariance and is defined as

$$\text{Correlation} = \frac{\text{Covariance (share return, market return)}}{\text{share return standard deviation} \times \text{market return standard deviation}}$$

This will give a value between -1 and +1 and is called the **correlation coefficient**.

If we square the correlation coefficient, we obtain the **coefficient of determination**, more commonly called R^2 which clearly ranges from 0 to 1 although sometimes the coefficient is given as a percentage – with or without the percentage sign (%), so some care is needed.

Using data from example 5 above

Example 7: Calculation of correlation and coefficient of determination

	Returns		Deviation from the mean		Deviation squared		Product of the deviations from the mean
	Company	Market	Company	Market	Company	Market	Company x Market
	16	10	4	(1)	16	1	(4)
	5	10	(7)	(1)	49	1	7
	32	16	20	5	400	25	100
	(8)	(24)	(20)	(35)	400	1225	700
	11	5	(1)	(6)	1	36	6
	20	11	8	0	64	0	0
	5	21	(7)	10	49	100	(70)
	9	27	(3)	16	9	256	(48)
	16	19	4	8	16	64	32
	14	15	2	4	4	16	8
TOTALS	**120**	**110**	**.0**	**0**	**1,008**	**1,724**	**731**

$$\text{Company SD} = \sqrt{\frac{1{,}008}{10}} = 10.04\% \qquad \text{Market SD} = \sqrt{\frac{1{,}724}{10}} = 13.13\%$$

$$\text{Covariance} = \frac{\text{Product of the deviations from the mean}}{\text{Number of measurements}} = \frac{731}{10} = 73.1\%$$

$$\text{Correlation coefficient} = \frac{\text{Covariance}}{\text{Company SD} * \text{Market SD}} = \frac{73.1}{10.04 * 13.13} = 55.45\%$$

Coefficient of determination = R^2 = 0.31

In this case R^2 explains how much of the share return is determined by the market return. The balance of the return ($1-R^2$) is explained by firm specific risk. This is diversifiable and should therefore be unrewarded. (Although this is a common interpretation, it is a little simplistic. We may be able to find a much better non-linear relationship between the market and share returns – in which case the firm specific risk would be less significant.)

One use of the coefficient of determination is to test the goodness of fit of the model. A value of one indicates a perfect fit, and therefore, a very reliable model for future forecasts. A value of zero, on the other hand, would indicate that the model fails to accurately model the dataset.

R^2 is also commonly used when considering fund performance (particularly in the US). An R^2 value of 1 means that all movements in the value of a fund are completely explained by movements in the relevant market index. Values between 0.85 and 1 indicate that the fund's performance patterns have been in line with the index.

Typically the R^2 value for the beta of a single share will be much lower. The median R^2 value for US corporations compared to the S&P 500 was 0.24 in 2008.

It is possible to calculate confidence intervals for the value of beta using the standard error of the beta estimate. The standard error is essentially the standard deviation of the actual data points from the regression line, since the actual points are given by the equation:

Actual Return, $r_i = A + \beta r_m + \text{error}$

A 95% confidence interval can be constructed as:

$\beta \pm 1.96 \times$ Standard Error

The range can be quite large and suggests that we should treat regression estimates of beta with caution, yet it will become apparent that many companies and analysts treat beta as a precise figure.

The relationship between beta and variability
We noted above that for all the data points:

Actual Return, $r_i = A + \beta r_m + \text{error}$

Therefore:

$\text{var}(r_i) = \text{var}(A) + \text{var}(\beta r_m) + \text{var}(\text{error})$

Now, if A and β are constants (which is generally assumed to be the case, at least in the short term), we can say that var(A) = 0 and $\text{var}(\beta r_m) = \beta^2 \text{var}(r_m)$.

So, $\text{var}(r_i) = \beta^2 \text{var}(r_m) + \text{var}(\text{error})$

Or , Firm Variability^2 = β^2(Market Variability)^2 + (Firm-Specific Risk)^2

= (Market Risk)^2 + (Firm-Specific Risk)^2

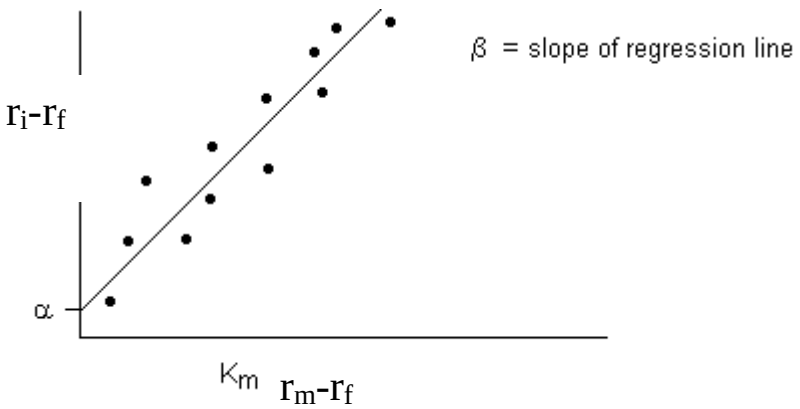
Example 8: Variability, market risk and specific risk
Rigidity in Balance SA has variability of 40% and a beta of 1.2. Market variability is 30%.

RIB's specific risk is √(1600 – (900 x 1.44)) = √304 = 17.43%.

RIB has relatively high market risk as denoted by a beta greater than 1 and relatively low specific risk.

7.3.2 The abnormal return
In the above regression we calculated beta by looking at the plot of share returns against market returns (the raw returns)

Instead we could have looked at *excess returns* as shown below



If the risk free rate remained constant throughout the period relating to the data points, then all the data points would simply be lower by r_f and beta would be the same. The likelihood is that beta would be similar using either method.

However α is easier to interpret in this scenario and is generally referred to as the **abnormal return** and can be either positive or negative. Alpha represents a return which is different to that implied by CAPM. Positive alphas imply returns over and above those predicted by CAPM whereas negative alphas imply under-performance. It is easiest to see this if we consider the special case when $r_m=r_f$, ie the market risk premium is zero. In these circumstances, CAPM implies that the cost of equity should be equal to the risk free rate, ie $r_i - r_f = 0$ with the regression line going through the origin. However, if α is positive, there is a super return, and the regression line is shifted upwards by α for all levels of risk exposure.

Example 9: Alpha
Careful Counters Contribution Club is an investment firm operating several different funds. One fund has the following performance history:

Year	Actual performance	Benchmark performance	Alpha
20x0	9%	8%	1%
20x1	10%	7.5%	2.5%
20x2	8%	8%	0%
20x3	7%	6%	1%

CCCC has achieved positive alpha over the whole period, except for 20x2, when the fund followed the market return.

Over the period, the risk free rate was around 3%, so the market premium ranged from 3% to 5%.

This analysis can be applied to funds as well as individual shares and many investors search out funds with high R^2 (that is, well diversified) and with positive abnormal returns. Since such funds are diversified, they should perform in line with the market and α should be zero, so any positive value represents an additional return over and above what should be expected.

8 Problems with beta

8.1 Commercial providers

There are many commercial beta services - the London Business School ("LBS"), Reuters and Bloomberg amongst others. Typically, providers of betas also provide values for R^2, the standard error and α, the abnormal return.

In general, beta providers calculate betas by comparing share price return against a market index (for share prices) rather than by looking at the return including dividends.

Does this matter? It depends. If the company you are looking at pays dividends in line with the market, probably not. However, if the company pays no dividends or has a very high dividend yield, then it will make some difference.

8.2 Adjusted betas

Most beta services also provide an adjusted beta which has the effect of pushing all betas towards 1. This is based on the observation that betas tend to 1 over time. Theoretically there is no justification for this, although it is possibly explained by acknowledging that as companies expand, they tend to diversify.

Bloomberg's adjusted beta, is calculated as follows:

Adjusted beta = Raw regression beta x 0.67 + 0.33

8.3 Beta estimation issues

There are 3 issues:

- Length of estimation period. The longer the period, the more data there is for the regression analysis which generally results in more reliable statistics. However, as the estimation period increases, the likelihood that the firm being analysed changes also increases – not just in terms of its business but also in terms of its gearing, a subject we will consider in a subsequent reading.
- The return interval. This is primarily a problem for shares with low turnover, i.e. where there is not much dealing in the company's stock. Data is available for a multitude of periods. However, if a daily interval is chosen for a stock with low turnover, there is a risk that returns will be reduced due to *nontrading bias*. In such cases, short intervals should not be used.
- The choice of market index. As we know, we should be using the market portfolio. However, there is no data for this. Ideally, a major index should be used which captures the whole market. The S&P 500 and the FTSE100 are commonly used. Although neither include all traded shares, they do cover a very high percentage by value for their particular exchange. However, when investors are making global investments, does it make sense to use a domestic index? This can be a particular problem where one or two companies dominate a market index.

Note too that this is another problem for R^2. The more dominant a company is relative to the index chosen, the higher the value for R^2 as the company's shares are more likely to perform in line with the index. This might imply low form-specific risk when the reality might be simply that the market index is inappropriate.

8.4 Other issues

We have already briefly suggested that linear regression may not be the most appropriate way of relating share returns to market returns. However, this is far from the only issue relating to the validity of regression betas.

If you have time (or interest) the further reading *2.1.1a Investment volatility: A critique of standard beta estimation and a simple way forward* makes some interesting observations about the calculation of beta. In particular, it suggests that the concept of α, the abnormal return so prized by some investors, is pure hokum.

For those interested in a deeper look at the concepts of correlation and the diversification of investments, read 2.1.1b Diversification: Should we be diversifying trends? This has implications in this topic and in risk management generally.

8.5 Forum discussion

Beta variation?

- **For those with access to Bloomberg (or a similar tool which allows beta analysis):**

Ideally for your own company (or if you do not work for a company with quoted shares, a company of your choosing) use Bloomberg to experiment with the beta calculation using different time periods, return intervals, indices, etc,

Post up the range of outcomes. If possible, also calculate 95% confidence intervals. Comment on your confidence in the results.

- **For those without access to Bloomberg:**

Research current betas (and other pertinent statistics) to establish the range.

Post up the range of outcomes to the module forum[3]: create a new post in the *Forum exercise from 2.1.1 Investor expectations (beta variation)* and title it U1/M2/2.1.1 Beta variation /[Your name]. If possible, also calculate 95% confidence intervals. Comment on your confidence in the results.

9 The cost of debt for the firm

The other major source of capital to the firm is debt and so we now need to understand its cost.

Unlike the cost of equity (broadly dividends and/or retained earnings), interest on corporate debt is generally tax deductible and this is the relevant cost for a firm. The post-tax cost of debt is given by:

$$K_d = i \times (1 - t_d)$$

Where i is the interest rate on debt and t_d is the corporate tax rate on debt interest.

Example 10: The cost of debt

Flexibility UN Ltd has a cost of debt of 5% and suffers a tax rate of 40%. All of its interest is allowable for tax and there are no brought forward losses.

The post tax cost of debt **5% x (1 – 40%) = 3%**

9.1 What rate of interest should be used?

The debt financing of the firm may include various debt instruments with differing tenors, ranking and security arrangements. Each of these will probably have a different cost. We can deal with this by weighting each interest rate in proportion to the total value of debt.

Additionally, as the cost of capital is used primarily to make decisions about the future which involve raising and investing new capital, we argue that we should focus on marginal costs, that is the cost of raising debt for the future.

Which rather begs the question: what amount and type of debt will the firm have in the future? Generally, in the absence of better information, we assume that the firm will operate with the same ratio of debt and equity over the medium term, re-financing debt as required (probably with the same debt instruments for the same tenor) and managing its equity with an appropriate dividend policy, and raising new equity if required.

Therefore, the relevant interest rate for each type of debt instrument might be that at which the firm can borrow given its present condition. This is relatively simply to determine if the firm has a traded bond – in which case the relevant interest rate would be the yield to maturity and

[3] You are expected to post forum activities to the appropriate forum within the two-week timeframe of the module. Please see the study timetable for reference.

its weighting in any calculation would be its market value. However, it is harder to determine if its bonds are not traded extensively or if it only has bank debt.

If the firm is rated, then current margins for similarly rated firms will be known. Even if the firm is not formally rated, an attempt may be made to estimate the rating. Similarly the pricing for a firm's credit default swaps can be used to estimate the current cost of its debt.

Other than traded debt instruments where there is a clear market value, the weighting used for other debt instruments will usually be taken as the accounting value of the debt instrument from the latest financial statements. For term debt, such figures may well be marked to market and include the impact of interest derivatives.

Note that in practice there are some occasions where it does not make sense to use future interest rates, especially where a firm has long term finance in place and where such financing is of a longer duration than the average investment project. Most firms do not trade in their own finance, but hold it to maturity, and using such interest rates still meets the main test - using interest rates which are appropriate for the (near) future.

9.2 What tax rate should be used?

This is not as straightforward as it is presented in many textbooks. Most large firms have operations and debt in various jurisdictions; this is true even when most external debt is raised centrally because such debt is usually on lent intragroup. So, the relevant tax rate depends on the jurisdiction where the debt is located.

Second, the tax rates in each jurisdiction should be a tax rate that applies in the future. In most cases, this will be the marginal rate in the relevant jurisdiction as opposed to the effective rate – the actual rate of tax paid on operating profits – as most treasurers will strive to ensure that debt is located in jurisdictions where they can achieve the maximum tax deduction. It should be noted here that the broad deductibility of interest in tax calculations is under threat. There are an increasing number of restrictions (which we can't go into in any detail) in many jurisdictions and the broad approach shown here may only properly apply for small levels of debt.

So, when we write $K_d = i \times (1 - t_d)$, this is really shorthand for:

$$K_d = \frac{\sum_1^n i_n \times (1-t_n) \times d_n}{\sum d_n}$$

Where d_n is the market value of each debt component if available, otherwise the accounting value from the latest financial statements, i_n is the rate of interest applicable to that debt component and t_n the relevant tax rate.

10 The weighted average cost of capital (WACC)

We have established that the cost of equity for a firm is the same as the equity return required by the investor and we have a mechanism to see what that cost is. We have also established the cost of debt.

However, most firms fund their investments with a mixture of equity and debt. In a later module we will consider the firm's capital structure, in particular the split between equity and debt. The important point at this juncture is not how a firm decides on the proportion of debt and equity, but that firms are generally funded by a mix of debt and equity.

So, firms need to measure the cost of this mix of equity and debt. We use a simple weighted average of the cost of equity and the cost of debt, the **Weighted Average Cost of Capital**, commonly referred to as the **WACC**.

The WACC is simply the weighted average of the cost of equity and the cost of debt.

The weights relate to the proportion of equity and debt used in financing the company. The shareholders, irrespective of when they acquired their shares, are all assumed to be willing investors at the current price. Therefore it is appropriate to weight the cost of equity by the market value of the shares. The same argument applies to debt instruments and, whilst these will not necessarily be true market values, it is common to state that the cost of debt is also weighted by the market value of debt, even though we are using proxies in some cases.

Therefore

$$\text{WACC} = \frac{(K_e \times E)+(K_d \times (1-t) \times D)}{E+D}$$

Where E = the market value of equity and D = the market value of debt.

A word of warning

The proportion of equity to debt, both of which should be market values when calculating the WACC, is often referred to as **gearing** or more precisely as **market gearing**. This should not be confused with traditional **book gearing** (which is usually also just referred to as gearing) which looks at the percentage value of book debt over book equity, the values in the financial statements.

Example 11: WACC

Columbo Inc, a US based corporation with operations in the US and UK. Columbo has an 8% Yankee bond with a face value of $300m trading at 103.28 paying an annual coupon and redeemable in 3 years and a term loan in the books at $124m which is pushed down to its UK operation. The term loan pays 3% over Prime, currently 3.5%. The tax rate is 24% in the US and 33% in the UK. Columbo's common stock trades on the NYSE at $1,042.25 and there are 99m shares in issue. The risk premium for S&P500 shares over the last 15 years has been 3.6% and the 10 year US Treasury rate is 4.2%. Columbo's beta is estimated at 1.3.

First, we calculate the yield to maturity (the internal rate of return) on the bond:

Year	Cash flow	
0	103.28	Bond market value
1	(8)	Coupon
2	(8)	Coupon
3	(108)	Coupon plus repayment at par
YTM	6.76%	(IRR of cash flows)

Then, we calculate the post-tax cost of debt, taking account of the market value of the bond and the jurisdiction of debt for tax purposes. Note that, in the absence of any better information, we have used the existing margin structure for the term loan.

$$K_d = \frac{6.76\% \times (1\text{-}24\%) \times \$300\text{m} \times 1.0328 + (3\% + 3.5\% \times (1\text{-}33\%) \times \$124\text{m}}{\$300\text{m} \times 1.0328 + \$124\text{m}}$$

K_d = 4.91%

Using CAPM, we calculate the cost of equity:

$$K_e = 4.2\% + 1.3 \times 3.6\% = 8.88\%$$

Therefore, the weighted average cost of capital is:

$$\text{WACC} = \frac{8.88\% \times \$1042.25 \times 99\text{m} + 4.91\% \times (\$300\text{m} \times 1.0328 + \$124\text{m})}{\$1042.25 \times 99\text{m} + \$300\text{m} \times 1.0328 + \$124\text{m}} = 8.86\%$$

Note that in the above calculation, the value for the cost of debt was already a post tax figure, so this is the figure we used in the WACC calculation.

11 Conclusion

You should now understand how the rate of return expected by a shareholder on his investment translates into the cost of equity for the firm and how the firm combines this with the cost of its debt in calculating its weighted average cost of capital.

In the following readings we will look at how the firm uses its WACC to assess investment opportunities.

You should also appreciate that beta and the Capital Asset Pricing Model are not precise tools. In the following readings we will consider further aspects which the corporate financier should be aware of.